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Exponential and Logarithmic Functions

The Richter Scale rating of earthquakes with a given intensity I is given by

$$\log_{10}\left(\frac{I}{I_0}\right)$$

where I_0 is the intensity of a certain (small) size earthquake. Find the Richter Scale rating of an earthquake having intensity $10,000,000I_0$.



11-1 ■ The exponential function

The exponential function

Expressions such as 2^x and $(1.05)^x$, where the exponent is a variable and the base is a constant, are important mathematical tools. These expressions can be used

1. to solve problems that involve the growth of bacteria in a culture, or the decay of a substance, and
2. to compute the amount to which a savings account increases when interest is compounded periodically.

We now wish to consider a function that is defined by an expression in which the exponent is a variable and the base is a constant. Such a function is called an **exponential function**.

Definition of exponential function

Given $b > 0$ and $b \neq 1$, the exponential function with base b is the function f defined by

$$f(x) = b^x$$

In our work with exponents thus far, we have defined 2^x when x is a rational number. That is, we know

$$2^0 = 1; \quad 2^2 = 4; \quad 2^{-2} = \frac{1}{2^2} = \frac{1}{4}; \quad 2^{-1/2} = \frac{1}{\sqrt{2}}; \quad 2^{1/3} = \sqrt[3]{2}$$

What we have not previously defined is

$$2^x \text{ (} x \text{ is irrational)}$$

For example, consider the value of

$$2^{\sqrt{2}}$$

The definition of $2^{\sqrt{2}}$ is beyond the scope of this text. For now, we assume that 2^x exists for all real numbers x , rational *and* irrational. We then assume that for $b > 0$, b^x is defined for all real numbers. We also assume that all properties of exponents previously learned extend to the real numbers.

Graph of the exponential function

Exponential functions can be graphed by finding several ordered pairs that belong to the function, plotting the points, and connecting the points with a smooth curve.

■ Example 11-1 A

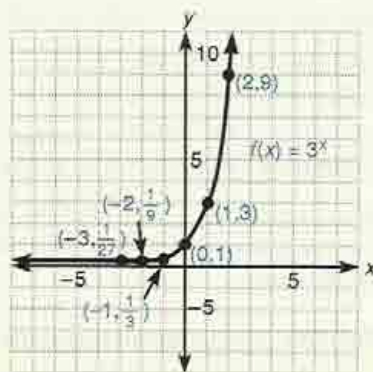
Sketch the graphs of the following exponential functions.

1. $f(x) = 3^x$

We make a table of related values.

x	-3	-2	-1	0	1	2
$y = f(x) = 3^x$	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$

Plot the points and connect them with a smooth curve.



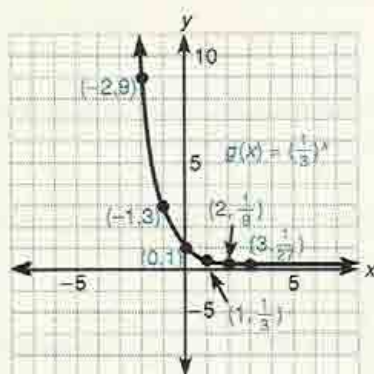
Note As x increases in value, $y = f(x)$ also increases in value.

2. $g(x) = \left(\frac{1}{3}\right)^x$

We make a table of related values.

x	-2	-1	0	1	2	3
$y = g(x)$ $= \left(\frac{1}{3}\right)^x$	$\left(\frac{1}{3}\right)^{-2} = 9$	$\left(\frac{1}{3}\right)^{-1} = 3$	$\left(\frac{1}{3}\right)^0 = 1$	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$	$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

Plot the graphs of the ordered pairs and draw a smooth curve.



Note As x increases in value, $y = g(x)$ decreases in value.

► **Quick check** Sketch the graph of $f(x) = 5^x$.

From our two graphs, we observe the following characteristics about the exponential function.

Characteristics of the exponential function

1. When $b > 1$, $f(x) = b^x$ increases in value as x increases in value. This is called **exponential growth**.
2. When $0 < b < 1$, $f(x) = b^x$ decreases in value as x increases in value. This is called **exponential decay**.
3. The graph of $f(x) = b^x$ passes through the point $(0, 1)$.
4. The graph of $f(x) = b^x$ approaches, but does not cross, the x -axis. The x -axis is an **asymptote**.

Note When $b = 1$, then $f(x) = 1^x = 1$ for every real number x , so we exclude $b = 1$. The graph is the horizontal line $y = f(x) = 1$.

In each case, we see that the domain of the function is the set of all real numbers, and since the graph of each function is entirely above the x -axis, the range is the set of positive real numbers.

Note The vertical line test for functions and the horizontal line test for one-to-one functions are both met. Thus the exponential function is a one-to-one function and so it has an inverse that is a function. We will study this inverse function in section 11-2.

The following property of exponents is used to solve some equations involving exponential expressions, called **exponential equations**.

Property of exponential equations

For any real numbers x , y , and b , $b > 0$, $b \neq 1$,
if $b^x = b^y$, then $x = y$

The following examples illustrate how this property is used to solve exponential equations.

Example 11-1 B

Find the solution set for the following exponential equations.

1. $3^x = 81$

Since $81 = 3^4$, then

$$3^x = (3^4)$$

$$x = 4$$

Replace 81 with 3^4

Property of exponential equations

The solution set is $\{4\}$.

2. $5^{4x+1} = (25)^{3x}$

Since $25 = 5^2$

$$5^{4x+1} = (5^2)^{3x}$$

$$= 5^2 \cdot 3x$$

$$= 5^{6x}$$

Replace $(25)^{3x}$ with $(5^2)^{3x}$

Property of exponents $(x^a)^b = x^{a \cdot b}$

$$\text{Thus, } 5^{4x+1} = 5^{6x}$$

$$\text{and } 4x + 1 = 6x$$

$$1 = 2x$$

$$x = \frac{1}{2}$$

Property of exponential equations

The solution set is $\left\{\frac{1}{2}\right\}$.

Quick check Find the solution set for $5^x = 125$.

We mentioned earlier that exponential equations occur in bacteria growth. The following example illustrates that growth.

Example 11-1 C

The number of bacteria in a culture that initially contains 3,000 bacteria triples every 15 hours. How many bacteria are there in the culture after (a) 15 hours, (b) 30 hours, (c) 45 hours? Write an equation for the number of bacteria in the culture, N , after t hours.

Since the bacteria *triples* (3 times) every 15 hours, after

- a. 15 hours, there are $3,000(3) = 9,000$ bacteria;
 b. 30 hours, the number of bacteria triples *twice*, so there are

$$[3,000(3)](3) = 3,000(3)^2 = 3,000(9) \\ = 27,000 \text{ bacteria}$$

- c. 45 hours, the number of bacteria triples *three* times, so there are

$$[3,000(3)](3)(3) = 3,000(3)^3 \\ = 3,000(27) = 81,000 \text{ bacteria}$$

In t hours, the number of times the bacteria triples is found by $\frac{t}{15}$, since it triples every 15 hours. Thus the number of bacteria, N , is given by

$$N = 3,000(3)^{t/15}$$

Mastery points

Can you

- Sketch the graph of an exponential function?
- Find the solution set of exponential equations?

Exercise 11-1

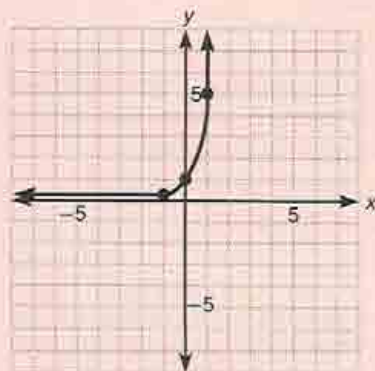
Sketch the graph of each given exponential function. See example 11-1 A.

Example $f(x) = 5^x$

Solution Choosing values of $x = -3, -2, -1, 0, 1, 2$, and 3 , we find corresponding values of $y = f(x)$.

x	-3	-2	-1	0	1	2	3
$y = 5^x$	$\frac{1}{125}$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125

Now graph the ordered pairs $\left(-3, \frac{1}{125}\right)$, $\left(-2, \frac{1}{25}\right)$, $\left(-1, \frac{1}{5}\right)$, $(0, 1)$, $(1, 5)$, $(2, 25)$, $(3, 125)$ and draw a smooth curve through the points.



- | | | | |
|---|----------------------|--|--|
| 1. $f(x) = 2^x$ | 2. $g(x) = 4^x$ | 3. $h(x) = \left(\frac{1}{2}\right)^x$ | 4. $f(x) = \left(\frac{1}{4}\right)^x$ |
| 5. $g(x) = 3^{-x}$ | 6. $h(x) = 2^{-x}$ | 7. $f(x) = 2^{2x}$ | 8. $g(x) = 3^{2x}$ |
| 9. $h(x) = \left(\frac{1}{3}\right)^{-x}$ | 10. $f(x) = 2^{x-1}$ | 11. $f(x) = 3^{x^2}$ | 12. $h(x) = 2^{x^2}$ |

Find the solution set of each exponential equation. See example 11–1 B.

Example $5^x = 125$

Solution Since $125 = 5^3$, then

$$\begin{array}{ll} 5^x = 5^3 & \text{Replace 125 with } 5^3 \\ x = 3 & \text{Property of exponential equations} \end{array}$$

The solution set is $\{3\}$.

- | | | | |
|--|---|---|--|
| 13. $2^x = 32$ | 14. $3^x = 27$ | 15. $4^x = 32$ | 16. $9^x = 243$ |
| 17. $16^x = 64$ | 18. $2^x = \frac{1}{16}$ | 19. $3^x = \frac{1}{9}$ | 20. $4^x = \frac{1}{64}$ |
| 21. $5^{-x} = 25$ | 22. $3^{-x} = 81$ | 23. $16^{-x} = 32$ | 24. $9^{2x} = 27$ |
| 25. $8^{3x} = 4$ | 26. $\left(\frac{1}{2}\right)^x = 16$ | 27. $\left(\frac{1}{3}\right)^x = 27$ | 28. $\left(\frac{2}{3}\right)^x = \frac{4}{9}$ |
| 29. $\left(\frac{4}{3}\right)^x = \frac{64}{27}$ | 30. $\left(\frac{1}{2}\right)^{-x} = 8$ | 31. $\left(\frac{3}{4}\right)^{-x} = \frac{64}{27}$ | 32. $(32)^{x-1} = 8$ |
| 33. $(27)^{2x+1} = 9$ | 34. $(25)^{4x+1} = 125$ | | |

See example 11–1 C.

35. The number of bacteria in a culture is initially 1,500 bacteria. If the number of bacteria triples every 12 hours, how many bacteria, N , are there in the culture after (a) 12 hours, (b) 36 hours, (c) 30 hours (use a calculator), (d) t hours?
36. The population of University City doubles every 10 years due to industries moving in. If the population was 40,000 in 1980, what is the population going to be in the year (a) 2000, (b) 2005, (c) t years from 1980?
37. Money deposited in a savings account will double every $\frac{72}{r}$ years, where r is the rate of interest per year. If \$5,000 is deposited at 8% interest, how much money is in the account after (a) 18 years, (b) 27 years, (c) t years?
38. A radioactive substance decays according to the equation
 $A = 200(3)^{-0.5t}$
 where A is the amount in grams and t is the time in months. How many grams of the substance are present after (a) 2 months, (b) 1 year?
39. Due to the introduction of an antibacterial substance into a culture that contains 8,000 bacteria, the bacteria are being destroyed according to the equation
 $N = 8,000(2)^{-t}$
 where N is the number of bacteria present and t is the time in hours. How many bacteria are present after (a) 3 hours, (b) 5 hours?
40. The production of an oil well is decreasing at a rate according to the equation
 $A = 1,000,000(2)^{-0.3t}$
 where A is the amount in barrels and t is the time in years. What is the production when (a) $t = 1$ year, (b) $t = 4$ years, (c) $t = 5$ years?



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Review exercises

1. Find the inverse of the function $f(x) = 4x - 3$.
See section 10-4.

2. Find the domain of the function $f(x) = \frac{4x}{3x - 5}$.
See section 10-1.

3. Given $f(x) = 2x - 1$ and $g(x) = x^2 + 2$, find $f[g(x)]$. See section 10-2.

Perform the indicated operations. See section 5-1.

4. $x^{2/3} \cdot x^{-1/2}$

5. $(x^3y^9)^{1/3}$

6. $\frac{x^{5/3}}{x}$

7. Find the solution set of the equation $z - \frac{5}{z} = 4$. See section 6-1.

11-2 ■ The logarithm

In section 11-1, we discussed the exponential function f as defined by $f(x) = b^x$, where $b > 0$ and $b \neq 1$. It is a one-to-one function and thus has an inverse that is a function. This inverse function is called the **logarithmic function**. Recall, to find the inverse function of a one-to-one function, we interchange the variables x and y . Then, given the exponential function with base b defined by $f(x) = b^x$, we replace $f(x)$ with y to get $y = b^x$ and interchange x and y to obtain the equation $x = b^y$. We introduce a new notation to define the logarithmic function.

Definition of logarithm

For every $b > 0$, $b \neq 1$,

$$x = b^y \text{ is equivalent to } y = \log_b x$$

In the definition,

1. to say that $x = b^y$ is equivalent to $y = \log_b x$ is to say that a solution for one equation is a solution for the other equation;
2. we restrict values of b to greater than 0 since, if $b < 0$, then b^x could not be defined for values of x such as $\frac{1}{2}$. Also, if $b = 1$, then $b^x = b^y$ for every real number value of x and y .

Note The word "log" is the abbreviation of the word **logarithm**. The notation $\log_b x$ is read "the logarithm of x to the base b " or, in shortened form, "log base b of x ."

Graph of the logarithmic function

We saw in chapter 10 that the graph of an inverse function f^{-1} is the reflection of the graph of function f with respect to the line $y = x$. We can then sketch the graph of the general logarithmic function $f^{-1}(x) = \log_b x$, using the graph of the general exponential function, $f(x) = b^x$ ($b > 0$, $b \neq 1$). (See figure 11-1.)

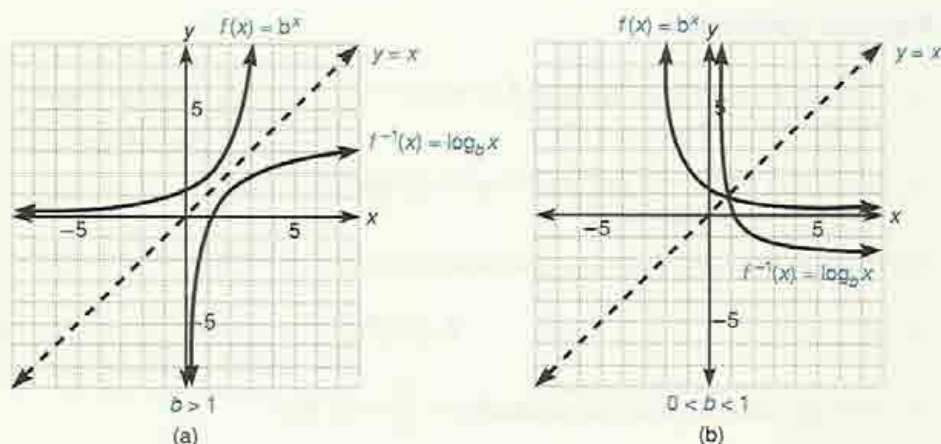


Figure 11-1

From the graphs in figure 11-1, we can determine that the logarithmic function has

1. domain = $\{x | x > 0\}$
2. range = $\{y | y \in \mathbb{R}\}$

Since, by definition, $x = b^y$ is equivalent to $y = \log_b x$, we use this relationship to write the statements in example 11-2 A.

■ Example 11-2 A

1. $8 = 2^3$ is equivalent to $3 = \log_2 8$.
2. $\frac{1}{27} = \left(\frac{1}{3}\right)^3$ is equivalent to $3 = \log_{1/3} \left(\frac{1}{27}\right)$.
3. $\frac{1}{16} = 4^{-2}$ is equivalent to $-2 = \log_4 \left(\frac{1}{16}\right)$.
4. $v = b^u$ is equivalent to $u = \log_b v$.

► **Quick check** Fill in the blanks.

$9 = 3^2$ is equivalent to _____

_____ is equivalent to $-2 = \log_7 \left(\frac{1}{49}\right)$.

$\frac{1}{27} = 3^{-3}$ is equivalent to _____.

_____ is equivalent to $-3 = \log_{1/5}(125)$.

Note It is important to remember that the **logarithm of a number to base b is the exponent to which the base b must be raised to get that number.** Thus **the logarithm of a number is an exponent.**

Using the previous equivalencies, we can state the following properties of the logarithmic function.

Properties of the logarithmic function

1. Since $b^1 = b$, then $\log_b b = 1$.
2. Since $b^0 = 1$, then $\log_b 1 = 0$.
3. Since $x = b^y$ is equivalent to $y = \log_b x$, then $b^{\log_b x} = x$.
4. Since $\log_b y = x$ means $y = b^x$, then $\log_b b^x = x$.

Example 11-2 B

1. $\log_3 3 = 1$ (property 1)
2. $\log_{1/4} \left(\frac{1}{4} \right) = 1$ (property 1)
3. $\log_6 1 = 0$ (property 2)
4. $\log_{1/2} 1 = 0$ (property 2)
5. $5^{\log_5 9} = 9$ (property 3)
6. $4^{\log_4 7} = 7$ (property 3)
7. $\log_2 2^5 = 5$ (property 4)
8. $\log_6 6^7 = 7$ (property 4)

Logarithmic equations

We can use the definition $x = b^y$ is equivalent to $y = \log_b x$ to find the solution set of equations involving logarithms, called **logarithmic equations**. The following examples illustrate how this is done.

Example 11-2 C

Find the solution set for each logarithmic equation.

1. $\log_4 64 = x$

By definition, $\log_4 64 = x$ is equivalent to $4^x = 64$.

Since $64 = 4^3$

$4^x = (4^3)$

Replace 64 with 4^3

$x = 3$

Property of exponential equations

Thus the solution set is $\{3\}$.

2. $\log_{10} x = 5$

By definition, $\log_{10} x = 5$ is equivalent to $10^5 = x$.

Since $10^5 = 100,000$

$x = (100,000)$

Replace 10^5 with 100,000

The solution set is $\{100,000\}$.

3. $\log_x 512 = 3$

By definition, $\log_x 512 = 3$ is equivalent to $x^3 = 512$.

Since $512 = 8^3$

$x^3 = (8^3)$

Replace 512 with 8^3

$x = 8$

Take principal cube root of each member

Thus the solution set is $\{8\}$.

► **Quick check** Find the solution set for $\log_x(256) = 4$.

Mastery points**Can you**

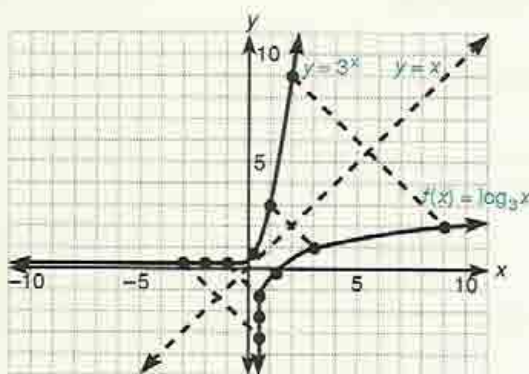
- Sketch the graph of a logarithmic function?
- Given an exponential equation, write the equivalent logarithmic equation?
- Given a logarithmic equation, write the equivalent exponential equation?
- Evaluate a logarithmic expression?
- Find the solution set of a logarithmic equation?

Exercise 11-2

Sketch the graph of $f(x) = \log_b x$ using the graph of $y = b^x$.

Example $f(x) = \log_3 x$

Solution We sketch the graph of $y = 3^x$. Because $g(x) = 3^x$ and $f(x) = \log_3 x$ are inverse functions, we reflect this graph with respect to the line $y = x$.



1. $f(x) = \log_2 x$

2. $g(x) = \log_4 x$

3. $h(x) = \log_5 x$

4. $F(x) = \log_{10} x$

5. $G(x) = \log_{1/2} x$

6. $H(x) = \log_{1/3} x$

Write the following exponential equations in logarithmic form. See example 11-2 A.

Example $9 = 3^2$

Solution $9 = 3^2$ is equivalent to $2 = \log_3 9$.

7. $81 = 3^4$

8. $16 = 4^2$

9. $64 = 2^6$

10. $512 = 8^3$

11. $\frac{1}{8} = \left(\frac{1}{2}\right)^3$

12. $\frac{1}{81} = \left(\frac{1}{3}\right)^4$

13. $\frac{8}{27} = \left(\frac{3}{2}\right)^{-3}$

14. $\frac{36}{25} = \left(\frac{5}{6}\right)^{-2}$

Write the following logarithmic equations in exponential form. See example 11-2 B.

Example $-2 = \log_7\left(\frac{1}{49}\right)$

Solution $-2 = \log_7\left(\frac{1}{49}\right)$ is equivalent to $\frac{1}{49} = 7^{-2}$

15. $\log_2 16 = 4$

16. $\log_3 81 = 4$

17. $\log_{10} 1,000 = 3$

18. $\log_5 625 = 4$

19. $\log_4\left(\frac{1}{16}\right) = -2$

20. $\log_2\left(\frac{1}{32}\right) = -5$

21. $\log_{10} 0.0001 = -4$

22. $\log_{10} 0.00001 = -5$

23. $\log_{1/2} 8 = -3$

24. $\log_{1/3} 81 = -4$

Evaluate each logarithmic expression.

Example $\log_{10} 1,000$

Solution Since $\log_{10} 1,000 = y$ means $(10)^y = 1,000$, we write both members with the same base. Now $1,000 = (10)^3$, so we have the equation

$$(10)^y = (10)^3$$

and $y = 3$

Thus $\log_{10} 1,000 = 3$

25. $\log_2 32$

26. $\log_3 81$

27. $\log_6 25$

28. $\log_7 1$

29. $\log_2\left(\frac{1}{4}\right)$

30. $\log_4\left(\frac{1}{64}\right)$

31. $\log_6\left(\frac{1}{216}\right)$

32. $\log_5\left(\frac{1}{25}\right)$

33. $\log_8 8$

34. $\log_{1/2}\left(\frac{1}{8}\right)$

35. $\log_{1/3}\left(\frac{1}{81}\right)$

36. $\log_{3/2}\left(\frac{27}{8}\right)$

37. $\log_{5/4}\left(\frac{125}{64}\right)$

38. $\log_5 \sqrt{5}$

39. $\log_7 \sqrt[3]{7}$

Find the solution set of each logarithmic equation. See example 11-2 C.

Example $\log_x(256) = 4$

Solution By definition, $\log_x(256) = 4$ is equivalent to $x^4 = 256$

Since $256 = 4^4$

$$x^4 = 4^4$$

Replace 256 with 4^4

$$x = 4$$

Take the principal 4th root of each member

The solution set is $\{4\}$.

40. $\log_x 25 = 2$

41. $\log_x 32 = 5$

42. $\log_x 243 = 3$

43. $\log_x\left(\frac{1}{27}\right) = -3$

44. $\log_x\left(\frac{1}{64}\right) = -6$

45. $\log_x 7 = 1$

46. $\log_x 5 = 1$

47. $\log_b 1 = 0$

48. $\log_{10} x = -2$

49. $\log_3 x = 4$

50. $\log_6 x = 3$

51. $\log_6 x = -2$

52. $\log_7 x = -3$

53. $\log_{1/4} x = 4$

54. $\log_2 x = 0$

55. $\log_b 1 = 0$

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56. For what values of x is $\log_6(x - 2)$ defined? (*Hint:* The domain of the logarithmic function is the set of positive real numbers.)

58. For what values of x is $\log_8(x^2 - 1)$ defined?

57. For what values of p is $\log_7(p + 1)$ defined?

59. For what values of y is $\log_9(y^2 - y - 12)$ defined?

Review exercises

1. Given $C = \frac{5}{9}(F - 32)$, find C when $F = 77$.

See section 1-5.

Perform the indicated operations. Assume all denominators are nonzero. See sections 4-2 and 4-3.

2. $\frac{25 - y^2}{2y + 3} \div (y + 5)$

3. $(3z + 2) - \frac{z + 4}{z - 2}$

4. Simplify $\left(\frac{ab^0}{c^{-3}}\right)^{-4}$ and eliminate negative exponents.

5. Simplify $\frac{3(2 - 5) - 3^2 + 8}{(-3)(-5)}$. See section 1-4.

See section 3-3.

6. Find the solution set of the equation $\sqrt[3]{3x - 1} + 1 = 0$. See section 6-5.

11-3 ■ Properties of logarithms

Logarithms have been used to simplify complex numerical computations. In this section, we shall see how logarithms can be used to perform these computations.

Product property of logarithms

If $b > 0$, $b \neq 1$, and x and y are positive real numbers, then

$$\log_b(xy) = \log_b x + \log_b y$$

Concept

The logarithm of the product of two positive real numbers is equal to the sum of the logarithms of the numbers. (Base remains same.)

To prove this property, we use the fact that a *logarithm of a number is an exponent*. Let $\log_b x = m$ and $\log_b y = n$, then,

$$b^m = x \text{ and } b^n = y$$

Using substitution,

$$x \cdot y = b^m \cdot b^n = b^{m+n} \quad \text{Product property of exponents}$$

By definition of logarithms,

$$xy = b^{m+n} \text{ is equivalent to } \log_b(xy) = m + n$$

Substituting $\log_b x$ for m and $\log_b y$ for n ,

$$\log_b(xy) = \log_b x + \log_b y$$

Example 11-3 A

Use the product property of logarithms to write each logarithmic statement as the sum of logarithms or as the logarithm of a single number.

- $\log_3(7 \cdot 4) = \log_3 7 + \log_3 4$
- $\log_5 6 + \log_5 9 = \log_5(6 \cdot 9) = \log_5 54$
- $$\begin{aligned} \log_2 20 &= \log_2(4 \cdot 5) = \log_2(2 \cdot 2 \cdot 5) \\ &= \log_2 2 + \log_2 2 + \log_2 5 && \text{Product property} \\ &= 1 + 1 + \log_2 5 && \log_b b = 1 \\ &= 2 + \log_2 5 && \text{Combine} \end{aligned}$$

Note We factored $20 = 4 \cdot 5 = 2 \cdot 2 \cdot 5$ since the base is 2 and $\log_2 2 = 1$ by property. We used prime factors.

► **Quick check** Write $\log_6 18$ as a sum of logarithms.

Note A common error is to assume that $\log_b(x \pm y)$ is the same as $\log_b x \pm \log_b y$. **This is not true.** To illustrate,

$$\log_2(1 + 1) = \log_2 2 = 1$$

while $\log_2 1 + \log_2 1 = 0 + 0 = 0$. Thus, $\log_2(1 + 1) \neq \log_2 1 + \log_2 1$.

The following two properties state the remaining properties of the logarithms. Their proofs are similar to the one just completed and they are left as exercises.

Quotient property of logarithms

If $b > 0$, $b \neq 1$, and x and y are positive real numbers, then

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

Concept

The logarithm of the quotient of two positive real numbers is equal to the logarithm of the numerator minus the logarithm of the denominator. (Base remains same.)

Example 11-3 B

Write each logarithmic expression as the difference of two logarithms or as the logarithm of a single number.

- $\log_6 \left(\frac{8}{9} \right) = \log_6 8 - \log_6 9$ Quotient property
- $\log_3 5 - \log_3 7 = \log_3 \left(\frac{5}{7} \right)$ Quotient property
- $$\begin{aligned} \log_5 \left(\frac{5}{8} \right) &= \log_5 5 - \log_5 8 && \text{Quotient property} \\ &= 1 - \log_5 8 && \text{Replace } \log_5 5 \text{ with } 1 \quad (\log_b b = 1) \end{aligned}$$

► **Quick check** Write $\log_2 5 - \log_2 3$ as a single number.

Power property of logarithms

If $b > 0$, $b \neq 1$, r is any real number and x is a positive real number, then

$$\log_b(x^r) = r \log_b x$$

Concept

The logarithm of any real power of a positive real number is equal to the exponent times the logarithm of the positive real number. (Base remains same.)

Example 11-3 C

Write each logarithmic expression as a constant times a logarithm or as the logarithm of a single number.

- $\log_3 2^3 = 3 \log_3 2$ Power property
- $6 \log_5 2 = \log_5 (2^6)$ Power property
 $= \log_5 64$ Replace 2^6 with 64 ($2^6 = 64$)
- $\log_7 (7^2) = 2 \log_7 7$ Power property
 $= 2 \cdot 1$ Replace $\log_7 7$ with 1 ($\log_b b = 1$)
 $= 2$

Note $(\log_b x)^2 \neq \log_b x^2$ because

$$(\log_b x)^2 = (\log_b x) \cdot (\log_b x)$$

$$\text{while } \log_b x^2 = \log_b (x \cdot x) = \log_b x + \log_b x$$

Using these properties and the following property of logarithms, we can find the solution set of other logarithmic equations.

Property of logarithms

If $x > 0$, $y > 0$, and $\log_b x = \log_b y$, then $x = y$ ($b > 0$, $b \neq 1$)

Example 11-3 D

Find the solution set of the following logarithmic equations.

1. $\log_5 (x - 2) = 3$

Writing this equation in its equivalent exponential form,

$$\begin{aligned} x - 2 &= 5^3 & \log_5 (x - 2) = 3 \text{ is equivalent to } x - 2 &= 5^3 \\ x - 2 &= 125 \\ x &= 127 \end{aligned}$$

The solution set is $\{127\}$.

2. $\log_2 x + \log_2 (x + 2) = 3$

Since the bases are the same, we can use the property

$$\log_b (xy) = \log_b x + \log_b y.$$

$$\begin{aligned} \log_2 x + \log_2 (x + 2) &= \log_2 x(x + 2) \\ &= \log_2 (x^2 + 2x) \end{aligned}$$

Now $\log_2 (x^2 + 2x) = 3$ is equivalent to $x^2 + 2x = 2^3$.

$$\begin{aligned}\text{Then } x^2 + 2x &= 8 \\ x^2 + 2x - 8 &= 0 \\ (x + 4)(x - 2) &= 0 \\ \text{Thus } x &= -4 \text{ or } x = 2.\end{aligned}$$

For $x = -4$, $\log_2(-4 + 2) = \log_2(-2)$, which is undefined since the domain must be positive. The solution set is $\{2\}$.

Note When solving logarithmic equations, we must always check our solutions for values that may not be in the domain of the function defined by the expressions in the original equation.

$$\begin{aligned}3. \log_3(6x - 7) + \log_3 x &= \log_3 5 \\ \log_3(6x - 7) + \log_3 x &= \log_3(6x - 7)x && \log_a x + \log_a y = \log_a(xy) \\ \log_3(6x - 7)x &= \log_3 5 && \text{Replace } \log_3(6x - 7) + \log_3 x \text{ with } \log_3(6x - 7)x \\ (6x - 7)x &= 5 && \text{Property of logarithms} \\ 6x^2 - 7x &= 5 \\ 6x^2 - 7x - 5 &= 0 \\ (3x - 5)(2x + 1) &= 0 && \text{Factor the left member} \\ 3x - 5 = 0 \text{ or } 2x + 1 = 0 \\ x = \frac{5}{3} & \quad x = -\frac{1}{2}\end{aligned}$$

Since the domain of the logarithm function allows only positive numbers, $x = -\frac{1}{2}$ is not possible. The solution set is $\left\{\frac{5}{3}\right\}$.

► **Quick check** Find the solution set of $\log_3(x + 6) - \log_3(x - 2) = 2$.

Mastery points

Can you

- Write a logarithmic expression in alternate forms using the properties?
- Solve logarithmic equations using the properties
 1. $\log_b(xy) = \log_b x + \log_b y$?
 2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$?
 3. $\log_b x^r = r \log_b x$?
 4. If $\log_b x = \log_b y$, then $x = y$?

Exercise 11-3

Use the properties of logarithms to write the following logarithms as a sum or difference of two, or more, logarithms, or as a product of a real number times a logarithm or as the sum or difference of a real number and a logarithm. All variables represent positive real numbers. See examples 11-3 A and B.

Example $\log_6 18$

$$\begin{aligned}\text{Solution } \log_6 18 &= \log_6(6 \cdot 3) \\ &= \log_6 6 + \log_6 3 \\ &= 1 + \log_6 3\end{aligned}$$

Factor 18 with base 6 as one factor
Product property of logarithms
 $\log_6 6 = 1$

1. $\log_b(3 \cdot 5)$ 2. $\log_2(7 \cdot 5)$ 3. $\log_3\left(\frac{7}{13}\right)$ 4. $\log_7\left(\frac{23}{17}\right)$
 5. $\log_b(5^3)$ 6. $\log_6(11^2)$ 7. $\log_{10}(5^4)$ 8. $\log_b(24)$
 9. $\log_{1/2}(36)$ 10. $\log_{10}\left(\frac{15}{16}\right)$ 11. $\log_5\left(\frac{24}{25}\right)$ 12. $\log_b\left(\frac{x^3}{y^2}\right)$
 13. $\log_b(xy)^3$ 14. $\log_b \sqrt[3]{x^2}$ 15. $\log_b(\sqrt{x} \cdot \sqrt[3]{y})$ 16. $\log_b\left(\frac{\sqrt[3]{x} \cdot \sqrt[4]{y}}{z}\right)$
 17. $\log_b(2x + 3y)$

Write the following logarithms as the logarithm of a single number or expression. Assume that all variables are positive real numbers. See examples 11–3 A, B, and C.

Example $\log_2 5 - \log_2 3$

Solution $\log_2 5 - \log_2 3 = \log_2\left(\frac{5}{3}\right)$ Quotient property of logarithms

18. $\log_4 5 + \log_4(11)$ 19. $\log_3(17) + \log_3 5$ 20. $\log_{10}(25) - \log_{10}(12)$
 21. $\log_7(11) - \log_7(16)$ 22. $2 \log_5(11)$ 23. $3 \log_6 4$
 24. $2 \log_2 7 + \log_2 5$ 25. $3 \log_{10} 3 + 2 \log_{10} 4$ 26. $4 \log_9 2 - \log_9(10)$
 27. $\frac{1}{5} \log_4(32) + \frac{1}{4} \log_4(81)$ 28. $\log_5 4 + 2 \log_5 3 - 3 \log_5 2$ 29. $2 \log_{10} 5 - \frac{1}{2} \log_{10} 9 + 3 \log_{10} 2$
 30. $4 \log_2 3 - \frac{1}{3} \log_2 8 + \log_2 7 - 2 \log_2 5$ 31. $\frac{1}{3} \log_b(x^2)$
 32. $\frac{1}{2} \log_b x + \frac{1}{2} \log_b y$ 33. $\frac{1}{4} \log_b(x^3) - \frac{1}{4} \log_b y$
 34. $\log_{10}(x + 3) + \log_{10}(x - 4)$ 35. $\log_5(x + 6) - 2 \log_5(3x) + \log_5(x - 4)$
 36. $\log_3(x - 7) + \log_3(2x + 1) - 3 \log_3(4x)$

Given $\log_b 2 = 0.3010$, $\log_b 3 = 0.4771$, $\log_b 7 = 0.8451$, and $\log_b 10 = 1$, compute the following logarithms and then write the statement in equivalent exponential form.

Example Given $\log_b 2 = 0.3010$ and $\log_b 3 = 0.4771$, find $\log_b 6$.

Solution Since $\log_b 6 = \log_b(2 \cdot 3) = \log_b 2 + \log_b 3$, then we replace $\log_b 2$ by 0.3010 and $\log_b 3$ by 0.4771 to obtain

$$\begin{aligned}\log_b 6 &= \log_b(2 \cdot 3) \\ &= \log_b 2 + \log_b 3 = 0.3010 + 0.4771 = 0.7781\end{aligned}$$

Thus $\log_b 6 = 0.7781$.

37. $\log_b(14)$ 38. $\log_b(20)$ 39. $\log_b(49)$ 40. $\log_b 8$ 41. $\log_b(42)$
 42. $\log_b\left(\frac{27}{4}\right)$ 43. $\log_b\left(\frac{7}{10}\right)$ 44. $\log_b \sqrt[3]{3}$ 45. $\log_b \sqrt{7}$ 46. $\log_b \sqrt{\frac{3}{4}}$
 47. $\log_b \sqrt[4]{9}$ 48. In exercises 37–47, what is b ?

Find the solution set of the following logarithmic equations. See example 11-3 D.

Example $\log_3(x + 6) - \log_3(x - 2) = 2$

Solution Using the property $\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$

$$\log_3(x + 6) - \log_3(x - 2) = \log_3 \left(\frac{x + 6}{x - 2}\right)$$

Thus,

$$\log_3 \left(\frac{x + 6}{x - 2}\right) = 2$$

$$\frac{x + 6}{x - 2} = 3^2$$

$$\frac{x + 6}{x - 2} = 9$$

$$x + 6 = 9x - 18$$

$$24 = 8x$$

$$x = 3$$

Replace $\log_3(x + 6) - \log_3(x - 2)$ with $\log_3 \frac{x + 6}{x - 2}$

$\log_3 \left(\frac{x + 6}{x - 2}\right) = 2$ is equivalent to $\frac{x + 6}{x - 2} = 3^2$

Multiply each member by $x - 2$

The solution set is $\{3\}$.

49. $\log_4 x + \log_4 5 = 3$

52. $\log_2 x - 3 \log_2 3 = 5$

55. $\log_3(x + 6) - \log_3(x - 2) = 2$

58. $\log_{10}(3x + 2) + \log_{10} 2 = 2$

61. $\log_2(x + 1) = \log_2 3 - \log_2(2x - 1)$

63. $\log_b(x + 2) + \log_b(2x - 1) = \log_b x$

65. $\log_b(6x - 5) = 2 \log_b x$

67. Prove that for $b > 0$, $b \neq 1$, and positive real numbers u and v ,

$$\log_b \left(\frac{u}{v}\right) = \log_b u - \log_b v$$

69. Show by an example that for positive real numbers u and v ,

$$\log_b(u + v) \neq \log_b u + \log_b v$$

50. $\log_3 x + \log_3 8 = 2$

53. $\log_3(2x + 1) - \log_3 5 = 3$

56. $\log_{10}(x) - \log_{10}(x + 3) = 1$

59. $\log_3 x + \log_3(x + 6) = 3$

62. $\log_5(x - 2) = \log_5 5 - \log_5(x + 3)$

64. $\log_2(2x - 1) = 2 \log_2 x$

66. $\frac{1}{2} \log_3 x = \log_3(x - 6)$

68. Prove that for $b > 0$, $b \neq 1$, positive real number u and any real number r ,

$$\log_b(u^r) = r \log_b u$$

51. $\log_5 x - 2 \log_5 2 = 1$

54. $\log_4(x - 5) - \log_4 3 = 2$

57. $\log_{10}(x + 21) + \log_{10} x = 2$

60. $\log_4(x - 15) + \log_4 x = 2$

Review exercises

What type of conic section figure do you get in the graph of the following equations? Find the x - and y -intercepts. See sections 9-1 to 9-3.

1. $x^2 + 3y^2 = 9$

2. $y = x^2 + x - 20$

3. $\frac{y^2}{9} - \frac{x^2}{16} = 1$

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4. Find the solution set of the equation $\frac{4}{z} - \frac{3}{z} = \frac{3}{4}$.
See section 4-7.
5. If y varies inversely as the square of x , find x when $y = 3$ if $y = 8$ when $x = 2$. See section 10-5.
6. Evaluate $(-27)^{2/3}$. See section 5-1.
7. Find the solution set of the inequality $|4 - 5x| \geq 2$.
See section 2-6.

11-4 ■ The common logarithms

We have worked thus far with logarithms to different bases. There are two logarithms that are most commonly used and are of great importance in mathematics. They are logarithms to base 10 and to base e , defined, respectively, by (1) $y = \log_{10} x$ and (2) $y = \log_e x$. Because 10 is the base of our number system, we will begin our discussion with base 10 logarithms. In section 11-5, we will discuss base e logarithms. The two bases have similar properties.

The function values of the logarithmic functions with base 10 are called the **common logarithms**. When we use 10 as a base, it is customary to omit the base when writing a logarithm. That is, we will use

$$\log x \text{ instead of } \log_{10} x, x > 0$$

and the base will be understood to be 10.

From our previous work with logarithms, we realize that a common logarithm of a number is the power to which 10 must be raised to obtain the number. For example,

$$\log 100 = 2 \quad \text{since} \quad 10^2 = 100$$

In like fashion,

$\log 1,000 = 3$	since $10^3 = 1,000$
$\log 100 = 2$	since $10^2 = 100$
$\log 10 = 1$	since $10^1 = 10$
$\log 1 = 0$	since $10^0 = 1$
$\log 0.1 = -1$	since $10^{-1} = \frac{1}{10} = 0.1$
$\log 0.01 = -2$	since $10^{-2} = \frac{1}{100} = 0.01$
$\log 0.001 = -3$	since $10^{-3} = \frac{1}{1,000} = 0.001$

Use of a calculator

To find the common logarithm of a number, we can use a calculator with a **log** key. The log key means \log_{10} , with base 10 being understood. To illustrate, to find $\log 724$, press

$$\boxed{7} \boxed{2} \boxed{4} \boxed{\log}$$

and read 2.8597386 on the display. Thus,

$$\log 724 \approx 2.8597 \text{ (correct to four decimal points)}$$

This means that $10^{2.8597} \approx 724$.

Note In the example, the logarithm of the number, 2.8597386, has two parts. We call

1. the integer number, 2, the **characteristic**, and
2. the decimal number, .8597386, the **mantissa** of the number.

■ Example 11-4 A

Find the following common logarithms using a calculator. Round to four decimal places.

1. $\log 715,000$

Using the calculator, press $\boxed{7} \boxed{1} \boxed{5} \boxed{0} \boxed{0} \boxed{0} \boxed{\log}$ and read 5.854306 on the display. Rounding off to four decimal places, $\log 715,000 \approx 5.8543$, and we have found $10^{5.8543} \approx 715,000$.

2. $\log 0.0749$

Press $\boxed{.} \boxed{0} \boxed{7} \boxed{4} \boxed{9} \boxed{\log}$ and read -1.125518 on the display. Thus $\log 0.0749 = -1.1255$ and $10^{-1.1255} \approx 0.0749$.

► **Quick check** Find the $\log 0.000315$

The antilogarithm

Now suppose we know the common logarithm of a number N and wish to determine N . That is, we know $\log N$ and wish to find N . The number N is called the **antilogarithm** (or abbreviated **antilog**) of $\log N$. Thus

$$\text{antilog } x = N \text{ is equivalent to } \log N = x$$

In example 11-4 A,

while $715,000$ is the antilog of 5.8543
 0.0749 is the antilog of -1.1255

We can use the calculator to find the antilogarithm of x . To find the antilogarithm on a calculator, we use either the $\boxed{10^x}$ key or the $\boxed{\text{INV}} \boxed{\log}$ key followed by the $\boxed{\log}$ key. To illustrate, to find the antilogarithm of 2.8597, we press

$$\boxed{2} \boxed{.} \boxed{8} \boxed{5} \boxed{9} \boxed{7} \boxed{10^x}$$

or

$$\boxed{2} \boxed{.} \boxed{8} \boxed{5} \boxed{9} \boxed{7} \boxed{\text{INV}} \boxed{\log}$$

and read 723.93571 on the display. Rounding off to three significant digits, we have found that

$$\log 724 \approx 2.8597$$

or

$$10^{2.8597} \approx 724$$

In our examples, we will use only the $\boxed{10^x}$ key.

■ Example 11-4 B

Find the antilogarithm of the following.

1. 3.8603

Press

$$\boxed{3} \boxed{.} \boxed{8} \boxed{6} \boxed{0} \boxed{3} \boxed{10^x}$$

and read 7249.376556 on the display. Thus the antilogarithm $\approx 7,250$ and we have found that

$$\log 7250 \approx 3.8603 \text{ and } 10^{3.8603} \approx 7250$$

2. -1.1255

Press

 $\boxed{1} \boxed{.} \boxed{1} \boxed{2} \boxed{5} \boxed{5} \boxed{+/-} \boxed{10^x}$

and read 0.074903135 on the display. Thus the antilogarithm ≈ 0.0749 and we have found that

$$\log 0.0749 \approx -1.1255 \text{ and } 10^{-1.1255} \approx 0.0749$$

► **Quick check** Find the antilogarithm of -3.5734 . ■

Mastery points

Can you

- Find the common logarithm of a number using a calculator?
- Find the antilogarithm of a number using a calculator?

Exercise 11-4

Find each of the following logarithms using a calculator. Then write the equivalent exponential statement. Round off to four decimal places. See example 11-4 A.

Example $\log 0.000315$

Solution Press $\boxed{.} \boxed{0} \boxed{0} \boxed{0} \boxed{3} \boxed{1} \boxed{5} \boxed{\log}$ and read -3.501689 on the display. Thus

$$\log 0.000315 \approx -3.5017$$

$$\text{and we have found that } 10^{-3.5017} \approx 0.000315$$

- | | | | |
|-------------------|----------------------|------------------------|-------------------|
| 1. $\log 8$ | 2. $\log 5.7$ | 3. $\log 53$ | 4. $\log 547$ |
| 5. $\log 80,200$ | 6. $\log 523,000$ | 7. $\log 794,000,000$ | 8. $\log 0.157$ |
| 9. $\log 0.00863$ | 10. $\log 0.0000941$ | 11. $\log 0.000000107$ | 12. $\log 0.0431$ |

Round off to four decimal places unless otherwise stated.

13. An artificial earth satellite is orbiting above the earth's surface at a height of 504,000 miles. Find $\log 504,000$.
14. The resistive force F exerted by water on a ship traveling at $1\frac{1}{2}$ statute miles per hour is 244,000 pounds. Find $\log 244,000$.
15. The angular velocity of the earth is 0.000073 radians per second and the earth's moment of inertia is 9.8×10^{37} kilograms per cubic meter. Find (a) $\log 0.000073$ and (b) $\log (9.8 \times 10^{37})$.
16. The current rate of interest on money market certificates is 6.75%. Find $\log (6.75\%)$. (*Hint: Write the percent in decimal form first.*)
17. Given the formula $H = w \log T$, find (a) H when $w = 110$ and $T = 360$ and (b) w when $H = 16.4$ and $T = 31.1$. (Round to two decimal places.)
18. Given $\log R = \frac{D}{10}$, find D when $R = 1.47$.
19. Given $P_H = -\log A_H$, find P_H when $A_H = 0.00167$.
20. The number of decibels of sound, N_{db} , in comparing the "loudness" of two sounds, is given by

$$N_{db} = 10 \log \left(\frac{P_1}{P_2} \right)$$
 where P_1 and P_2 are the signal power levels to be compared. Find N_{db} when $P_1 = 950$ and $P_2 = 25$.

21. Using exercise 20, find N_{db} when $P_1 = 1,070$ and $P_2 = 35$.
22. A decibel power gain, G_{db} , of an electronic amplifier is given by

$$G_{db} = 10 \log \left(\frac{P_0}{P_i} \right)$$

where P_0 = the power output and P_i = the power input. Find G_{db} when $P_0 = 20$ and $P_i = 0.005$.

23. Using exercise 22, find G_{db} when $P_0 = 18$ and $P_i = 0.006$.

Find the antilogarithm of each number, using a calculator. See example 11-4 B.

Example -3.5734

Solution Press $\boxed{3} \boxed{.} \boxed{5} \boxed{7} \boxed{3} \boxed{4} \boxed{+/-} \boxed{10^x}$ and read 0.0002671 on the display. Thus, the antilogarithm ≈ 0.000267 and we have found that

$$\log 0.000267 \approx -3.5734 \text{ and } 10^{-3.5734} \approx 0.000267$$

- | | | | |
|----------------------|---------------|-------------------|---------------|
| 24. 0.5416 | 25. 3.8445 | 26. 4.4997 | 27. 7.9138 |
| 28. -1.2472 | 29. -4.7825 | 30. 8.8340 | 31. 2.5283 |
| 32. -6.0104 | 33. -2.2301 | 34. -3.2391 | 35. -6.4727 |

Solve the following statements as indicated. Round off to four decimal places.

36. Given $P_H = -\log A_H$, find A_H when $P_H = 7.78$.
37. Given $H = w \log T$, find T when $w = 1.87$ and $H = 3.15$.

38. Given $\log R = \frac{D}{10}$, find R when $D = 44.5$.

39. Given $G_{db} = 10 \log \left(\frac{P_0}{P_i} \right)$, find P_0 when $G_{db} = 15$ and $P_i = 0.60$. (Hint: Use the property $\log \left(\frac{P_0}{P_i} \right) = \log P_0 - \log P_i$)

40. Given $z_0 = 276 \log \left(\frac{b}{a} \right)$, find (a) b when $z_0 = 487$ and $a = 0.112$ and (b) a when $z_0 = 552.1$ and $b = 5.7$.

41. A chemist defines the pH (hydrogen potential) of a solution by

$$\text{pH} = -\log[\text{H}^+]$$

where $[\text{H}^+]$ denotes the numerical value for the concentration of hydrogen ions in aqueous solution in moles per liter. Find the pH of the solution with hydrogen concentration, $[\text{H}^+]$, as 5.0×10^{-4} .

42. Using exercise 41, find pH with the hydrogen concentration 6.1×10^{-7} .

- 43.** Using exercise 41, find the hydrogen ion concentration with a pH 5.8.

44. Using exercise 41, find the hydrogen ion concentration with a pH 6.8.

Review exercises

Simplify the following expressions.

- $4 - 2[12 - (7 - 5) - (13 + 2)]$ See section 1-4.
- $5ab - (4a + 2b - a) - (3b + ab)$ See section 1-6.
- $\sqrt{18} + \sqrt{50} - \sqrt{98}$ See section 5-5.
- Jan takes 24 minutes to do a particular job, and her sister Jody takes 30 minutes to do the same job. How long would it take the sisters to do the job working together? See section 4-8.
- $(2 - \sqrt{-36}) - (3 + \sqrt{-49})$ See section 5-7.
- One bottle of battery acid is a 10% solution, and another bottle is a 4% solution. How many cubic centimeters of each solution are necessary to make 100 cm³ of 8% solution? Use systems of linear equations. See section 8-2.

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11-5 ■ Logarithms to the base e **Natural logarithms**

In section 11-4, we mentioned two important logarithmic bases: base 10 and base e . Also in section 11-4, we studied the base 10 logarithms, called the **common logarithms**. In applications, logarithms to base e are more frequently used than logarithms to base 10. We call the base e logarithms the **natural logarithms**.

The number e is an irrational number, as is the irrational number π . Computed $e \approx 2.7182818284 \dots$, but we generally round off this value and use the value $e \approx 2.718$.

Just as a special symbol was used to represent the logarithm to base 10, that is, “log,” we also have a special symbol to represent the natural logarithm of positive number x . We denote

$$\log_e x \quad \text{by} \quad \ln x$$

Definition of the natural logarithm

$$\ln x = y \quad \text{is equivalent to} \quad e^y = x$$

Changing the base

To develop methods for finding decimal approximations of $\ln x$, we first develop a special formula for converting logarithms from one base to another. Since we can find the logarithm to the base 10 for any positive number, we will then be able to find the logarithm for any positive number to any other base, such as base e .

Change of base property

Let $a > 0$, $b > 0$, $x > 0$, $a \neq 1$, and $b \neq 1$. Then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

and using common logarithms,

$$\log_a x = \frac{\log x}{\log a}$$

■ **Example 11-5 A**

Find the value of each logarithm, using common logarithms. Round off to four decimal places.

1. $\log_5 13$

$$\log_5 13 = \frac{\log (13)}{\log (5)} \quad \text{Replace } x \text{ with } 13 \text{ and } a \text{ with } 5 \text{ in } \frac{\log x}{\log a}$$

On the calculator, press

$$\boxed{1} \boxed{3} \boxed{\log} \boxed{\div} \boxed{5} \boxed{\log} \boxed{=}$$

and read 1.5936926 on the display. Thus, $\log_5 13 \approx 1.5937$.

2. $\log_{14} 33.5$

$$\log_{14} 33.5 = \frac{\log(33.5)}{\log(14)}$$

Replace x with 33.5 and a with 14 in $\frac{\log x}{\log a}$

On the calculator, press

$$\boxed{3} \boxed{3} \boxed{.} \boxed{5} \boxed{\log} \boxed{\div} \boxed{1} \boxed{4} \boxed{\log} \boxed{=}$$

and read 1.330606 on the display. Thus, $\log_{14} 33.5 \approx 1.3306$.3. $\log_e 0.234$

$$\begin{aligned} \log_e 0.234 &= \frac{\log 0.234}{\log(e)} && \text{Replace } x \text{ with } 0.234 \text{ and } a \text{ with } e \text{ in } \frac{\log x}{\log a} \\ &= \frac{\log 0.234}{\log(2.718)} && \text{Replace } e \text{ with } 2.718 \end{aligned}$$

On the calculator, press

$$\boxed{.} \boxed{2} \boxed{3} \boxed{4} \boxed{\log} \boxed{\div} \boxed{2} \boxed{.} \boxed{7} \boxed{1} \boxed{8} \boxed{\log} \boxed{=}$$

and read -1.4525848 on the display.

$$\log_e 0.234 = \ln 0.234 \approx -1.4526$$

► **Quick check** Find the value of $\log_{16} 25.7$ using common logarithms. Round off to four decimal places. ■

Natural logarithms can be found directly on a calculator that has a key labeled $\boxed{\ln}$.

■ Example 11-5 B

Find the value of each natural logarithm correct to four decimal places, using the calculator.

1. $\ln 57.6$ Enter $\boxed{5} \boxed{7} \boxed{.} \boxed{6} \boxed{\ln}$ and read 4.0535225 on the display. Thus,

$$\ln 57.6 \approx 4.0535$$

2. $\ln 0.5731$

For numbers between 0 and 1, the natural logarithm will be negative as with common logarithms.

Enter $\boxed{.} \boxed{5} \boxed{7} \boxed{3} \boxed{1} \boxed{\ln}$ and read -0.5566951 on the display. Thus,

$$\ln 0.5731 \approx -0.5567$$

► **Quick check** Find the value of $\ln 0.932$, correct to four decimal places using the calculator. ■

In example 1, we found that $\ln 57.6 \approx 4.0535$. Suppose we want $\text{antiln } 4.0535$ (which is 57.6). We use the $\boxed{e^x}$ key and enter

$$\boxed{4} \boxed{.} \boxed{0} \boxed{5} \boxed{3} \boxed{5} \boxed{e^x}$$

into the calculator. Read 57.5987 on the display. Thus, $\text{antiln } 4.0535 \approx 57.6$ and

$$e^{4.0535} \approx 57.6$$

The properties of logarithms that we used with common logarithms also apply to the natural logarithms.

Growth and decay

Problems involving natural growth and natural decay require the use of the exponential and logarithmic functions. The general equations for natural growth and natural decay are given by

$$\begin{aligned}\text{growth, } q &= q_0 e^{rt} \quad (r > 0) \\ \text{decay, } q &= q_0 e^{-rt} \quad (r > 0)\end{aligned}$$

where q is the quantity (or number) present at time $t > 0$, q_0 is the quantity (or number) present when time $t = 0$, and r is the rate of growth or decay.

Note 1. e is the base of the natural logarithm.
2. We assume $r > 0$ at all times. Then $-r < 0$ and this causes the decrease in quantity.

We will use these equations to find the time it takes for a quantity q_0 at time $t = 0$ to grow or decay to a quantity q . Solving for t , we obtain the following equations:

Growth and decay formulas

$$t = \frac{1}{r} \ln\left(\frac{q}{q_0}\right), \text{ for growth; } t = -\frac{1}{r} \ln\left(\frac{q}{q_0}\right), \text{ for decay.}$$

Example 11-5 C

- How long will it take the earth's population to double if it continues to grow at the rate of 2 percent per year compounded continuously? Find the answer to the nearest tenth.

Since the earth's population is to double, we are given that $q = 2q_0$. Also we are given $r = 0.02$, and we want time t . Using the equation

$$t = \frac{1}{r} \ln\left(\frac{q}{q_0}\right)$$

$$t = \frac{1}{(0.02)} \ln\left(\frac{2q_0}{q_0}\right)$$

$$= 50 \ln 2$$

$$= 50(0.6931)$$

$$\approx 34.7$$

Replace r with 0.02 and q with $2q_0$

Reduce by q_0

Perform indicated operations

The population will double in approximately 34.7 years.

- Radioactive strontium 90 is used in nuclear reactors and decays according to the equation


$$q = q_0 e^{-0.0248t}$$

Find the half-life of strontium 90.

Note **Half-life** is the time necessary for a substance to decay to one-half of its original quantity.

We are given $q = \frac{1}{2}q_0$, $r = 0.0248$, and we want t in years. Using the equation

$$\begin{aligned}
 t &= -\frac{1}{r} \ln\left(\frac{q}{q_0}\right) \\
 t &= -\frac{1}{(0.0248)} \ln\left(\frac{\frac{1}{2}q_0}{q_0}\right) && \text{Replace } r \text{ with } 0.0248 \text{ and } q \text{ with } \frac{1}{2}q_0 \\
 &= -40.3 \ln\left(\frac{1}{2}\right) && \text{Reduce by } q_0 \\
 &= -40.3(-0.6931) && \text{Perform indicated operations} \\
 &\approx 27.9
 \end{aligned}$$

Therefore, the half-life of strontium 90 is approximately 27.9 years. 

Mastery points

Can you

- Find the logarithm of any positive number to any given base?
- Find the natural logarithm of a positive number using the common logarithms?
- Use natural logarithms in working applications of natural growth and natural decay?

Exercise 11-5

Find each of the following logarithms. Round off to four decimal places. See example 11-5 A.

Example $\log_{16} 25.7$

Solution $\log_{16} 25.7 = \frac{\log(25.7)}{\log(16)}$ Replace x with 25.7 and a with 16

On the calculator enter, $\boxed{2} \boxed{5} \boxed{.} \boxed{7} \boxed{\log} \boxed{\div} \boxed{1} \boxed{6} \boxed{\log} \boxed{=}$ and read 1.1709241 on the display. Therefore, $\log_{16} 25.7 \approx 1.1709$.

1. $\log_5 8$

2. $\log_7 9$

3. $\log_4 25$

4. $\log_7 47$

5. $\log_2 0.156$

6. $\log_3 0.0324$

$\boxed{7}$. $\log_{23} 45.6$

8. $\log_{18} 157$

See example 11-5 B.

Example $\ln 0.932$

Solution Using the calculator, press $\boxed{.} \boxed{9} \boxed{3} \boxed{2} \boxed{\ln}$ and read -0.0704225 on the display. Thus $\ln 0.932 \approx -0.0704$.

9. $\log_e 5$

$\boxed{10}$. $\log_e 3$

11. $\log_e 2.73$

$\boxed{12}$. $\log_e 107$

13. $\ln 6.73$

14. $\ln 5.13$

15. $\ln 0.0504$

16. $\ln 0.00619$

17. $\ln 347$

18. $\ln 197$

19. $\log 5e$

20. $\log 3e$

$\boxed{21}$. $\ln 7e$

22. $\log 6e^3$

23. $\ln 3e^3$

Find the antilog of the following logarithms.

24. 1.3415

25. 2.6137

26. 4.0076

27. -0.1234

28. -3.1743

29. -2.9145

30. -0.0421

Solve the following problems. Round off to the nearest tenth. See example 11-5 C.

31. When money is invested at interest rate r percent per year compounded continuously, after t years it is worth

$$q_0 e^{rt/100} \text{ dollars}$$

where q_0 is the amount invested. How long will it take for an investment of \$200,000 to triple at 6% per year compounded continuously?

(Hint: Use $q = q_0 e^{rt/100}$, where $r = 6$.)

32. Using exercise 31, how long will it take for \$15,000 to become one and one-half times as great?

33. Using the equation for the decay of strontium 90,

$$q = q_0 e^{-0.0248t}$$

how many years will it take until only one-fourth of the original amount of the substance is left?

34. Using the formula of exercise 33, how long will it take for 90% of the strontium 90 to decay?

35. Radioactive carbon 14 diminishes by radioactive decay according to the equation

$$q = q_0 e^{-0.000124t}$$

where t is in years. Carbon 14 enters all living tissue through carbon dioxide and is maintained to be constant as long as the plant or animal is alive. The decay takes place when the tissues are dead. Estimate the age of the skull uncovered if 20% of the original amount of carbon 14 is still present.

(Hint: $q = 0.2q_0$.)

36. Using the information of exercise 35, estimate the age of the skull if only 5% of the original amount of carbon 14 is still present.

37. For relatively clear bodies of water, light intensity is reduced according to the equation

$$I = I_0 e^{-kd}$$

where I is the intensity at d feet below the surface and I_0 is the intensity at the surface. In a particular body of water, $k = 0.00853$. Find the depth to the nearest tenth of a foot at which the light is reduced to 10% of that at the surface.

38. Using exercise 37, when $k = 0.0585$, find the depth at which the light is reduced to 2% of that at the surface.

39. Suppose a certain species of bees increases in number according to the exponential equation

$$q = 25e^{0.2t}$$

where t is measured in days. In how many days, correct to the nearest tenth, will there be 375 bees?

Review exercises

State the domain of the following functions. See section 10-1.

1. $f = \{(-4,3), (2,3), (0,-8), (1,10)\}$

3. Is f in exercise 1 a one-to-one function? If not, why not? See section 10-4.

2. $g(x) = \sqrt{2x-3}$

4. Find the solution set of the following system of equations:

$$x^2 + y^2 = 5$$

$$x - y = 1$$

See section 9-4.

Graph the following functions. See section 10-3.

5. $f(x) = 4x + 3$

6. $g(x) = x^2 - x + 6$

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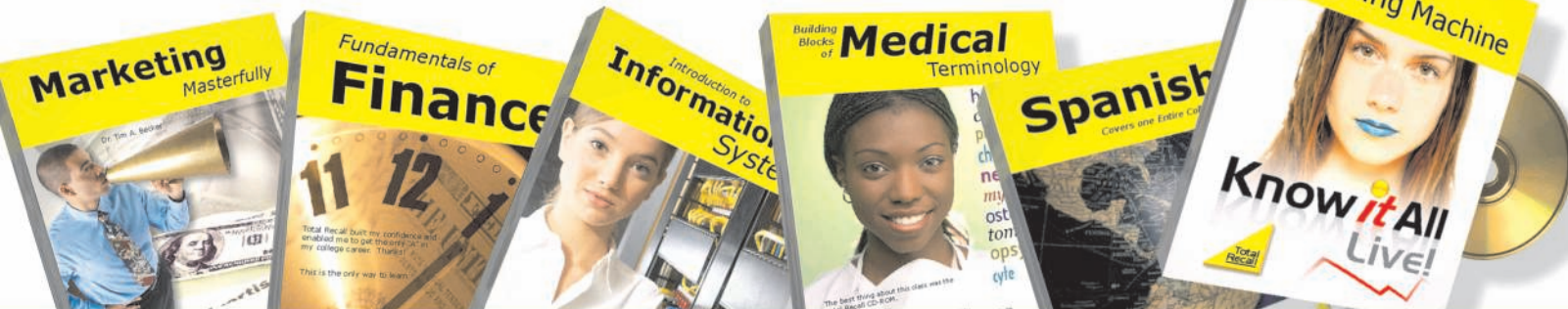
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11-6 ■ Exponential equations

In section 11-1, we found the solution set of some exponential equations, that is, equations in which the exponent is a variable. Now we consider some exponential equations whose solution set can be found by using logarithms. To do this, we use a property of logarithms.

Property of logarithms

Given positive real numbers x , y , and b , $b \neq 1$, $b > 0$,

$$\text{if } x = y, \text{ then } \log_b x = \log_b y \quad (1)$$

■ Example 11-6 A

Find the solution set of each exponential equation, correct to the nearest hundredth.

1. $3^{x+7} = 11$

$$\log(3^{x+7}) = \log(11)$$

Take the common logarithm of each member.

$$(x+7)\log 3 = \log(11)$$

Power property of logarithms

$$x+7 = \frac{\log(11)}{\log 3}$$

Divide each member by $\log 3$

$$x = \frac{\log(11)}{\log 3} - 7$$

Subtract 7 from each member

Using the calculator, enter

$$\boxed{1} \boxed{1} \boxed{\log} \boxed{\div} \boxed{3} \boxed{\log} \boxed{=} \boxed{-} \boxed{7} \boxed{=}$$

and read -4.8173417 on the display. Thus, $x \approx -4.82$ and the solution set is $\{-4.82\}$.

2. An amount of money P (called the principal) is invested at r percent compounded annually. The amount of money A after t years is given by the equation $A = P(1+r)^t$. How long will it take $P = \$1,000$ to be worth $A = \$2,500$ at 6% compounded annually?

We want t in years when $r = 0.06$, $A = \$2,500$, and $P = \$1,000$.

$$(2,500) = (1,000)[1 + (0.06)]^t \quad \text{Replace } r \text{ with } 0.06, A \text{ with } 2,500, \text{ and } P \text{ with } 1,000$$

$$2.5 = (1.06)^t$$

Divide each member by 1,000; combine

$$\log(2.5) = \log(1.06)^t$$

Take the log of each member

$$\log(2.5) = t \log(1.06)$$

Power property of logs

$$t = \frac{\log(2.5)}{\log(1.06)}$$

Divide each member by $\log(1.06)$

Using the calculator, enter

$$\boxed{2} \boxed{.} \boxed{5} \boxed{\log} \boxed{\div} \boxed{1} \boxed{.} \boxed{0} \boxed{6} \boxed{\log} \boxed{=} \quad \text{and read } 15.725209 \text{ on the display. Thus, } t \approx 15.73$$

Therefore an investment of \$1,000 would take approximately 15.73 years to grow to \$2,500 when compounded annually at 6%.

Note $t = \frac{\ln(2.5)}{\ln(1.06)}$ will work just as well.

► **Quick check** Find the solution set of $4^{x-2} = 21$, correct to the nearest hundredth.

Mastery points**Can you**

- Find the solution set of an exponential equation by using common logarithms (or natural logarithms)?
- Solve applications of exponential equations?

Exercise 11-6

Find the solution set of the given exponential equation. Find solutions correct to the nearest hundredth.

Example $4^{x-2} = 21$

Solution $\log(4^{x-2}) = \log(21)$

$$(x-2)\log 4 = \log(21)$$

$$x-2 = \frac{\log(21)}{\log 4}$$

$$x = \frac{\log(21)}{\log 4} + 2$$

Take the log of each member

Power property of logs

Divide each member by $\log 4$

Add 2 to each member

Using the calculator, enter

$\boxed{2}\boxed{1}\boxed{\log}\boxed{\div}\boxed{4}\boxed{\log}\boxed{=}\boxed{+}\boxed{2}\boxed{=}$ and read 4.1961587 on the display.

Thus, $x \approx 4.20$ and the solution set is $\{4.20\}$.

- $2^x = 9$
 - $7^x = 8$
 - $4^{x+1} = 6$
 - $5^{x-2} = 11$
 - $3^{-x} = 10$
 - $9^{-x} = 19$
 - $6^{2x-1} = 12$
 - $8^{3x+1} = 9$
 - $3^{x+1} = 4$
 - $5^{x+2} = 3$
 - $16^{1-x} = 13$
 - $19^{2-x} = 17$
 - $4^5 - 2x = 9$
 - $2^{x^2} = 7$
 - $\left(\frac{1}{3}\right)^{1-x} = 5$
 - $\left(\frac{3}{5}\right)^{2x} = 2$
- Solve the equation $y = x^{3n}$ for n using the common logarithms, \log .
 - Solve the equation $y = e^{kt}$ for t using the natural logarithms, \ln .
 - What rate of interest, to the nearest tenth of a percent, is required for \$50 to yield \$61.50 after 5 years, compounded annually? [Hint: Use $A = P(1+r)^t$.]
 - If \$2, compounded annually, yields \$2.98 after 8 years, what is the rate of interest, to the nearest hundredth of a percent?
 - If \$65 yields \$82.50 when the rate of interest is 6.25% compounded annually, how many years was the money invested? (to the nearest tenth)
 - For how many years, to the nearest hundredth, must \$1,050 be invested at 8.5% interest compounded annually to have a return of \$1,760?
 - A certain bacteria divides every 15 minutes to produce two new bacteria. If the number increases according to the equation $A = A_0 2^{4t}$ where t is in hours, how long would it take, to the nearest tenth of an hour, for 2,000 bacteria to increase to 750,000 bacteria?
 - Work exercise 23 for 5,000 bacteria to increase to 1,000,000 bacteria.
 - Using the formula of exercise 23, how long would it take for the bacteria to triple?
 - Radium disintegrates in such a way that of q_0 milligrams present at a given time, $q = q_0(0.96)^t$ milligrams will remain after t centuries. What is the half-life of radium, to the nearest tenth of a century?

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27. Using exercise 26, in how many centuries (to the nearest tenth) will 200 milligrams of radium disintegrate to 125 milligrams?
28. A machine depreciates according to the formula $V = V_0(1 - r)^t$ where V is the value after t years, V_0 is the original value, and r percent is the constant rate of depreciation. In how many years (to the nearest tenth) will the machine depreciate to one-half its original value at a depreciation rate of 15%?
29. Using exercise 28, in how many years (to the nearest hundredth) will the value depreciate from a value of \$6,500 to a value of \$5,000 at a depreciation rate of 12%?
30. Using exercise 28, what is the rate of depreciation (to the nearest tenth) if a machine depreciates to one-fourth its original value in 9 years?
31. Using exercise 28, what is the rate of depreciation (to the nearest tenth) if the machine reduces in value from \$1,600 to \$1,280 in 6 years?

Review exercises

Factor the following expressions. See section 3–8.

- $5x^2 - x - 4$
- $5x^3 - 2x^2 + x$
- Find the solution set of the quadratic-type equation $w^4 - 5w^2 = 14$. See section 6–6.
- Find the distance in the plane from $(-4, 2)$ to $(6, 1)$. What are the coordinates of the midpoint of the line segment with these endpoints? See section 7–2.
- $25z^2 - 36$
- $9z^2 - 12z + 4$
- Find the solution set of the rational inequality $\frac{x+5}{x-2} \leq 0$. See section 6–7.

Chapter 11 lead-in problem

The Richter Scale rating of earthquakes with a given intensity I is given by $\log_{10} \frac{I}{I_0}$, where I_0 is the intensity of a certain (small) size earthquake. Find the Richter Scale rating of an earthquake having intensity $10,000,000I_0$.

Solution

We want $\log_{10} \frac{I}{I_0}$ when $I = 10,000,000I_0$. Thus,

$$\begin{aligned} \text{Richter rating} &= \log_{10} \frac{10,000,000I_0}{I_0} && \text{Replace } I \text{ with } 10,000,000I_0 \\ &= \log_{10} 10,000,000 && \text{Reduce by } I_0 \\ &= \log_{10} 10^7 && \text{Write } 10,000,000 \text{ as exponential form } 10^7 \\ &= 7 \end{aligned}$$

The Richter Scale rating of the earthquake was 7.

Chapter 11 summary

- An **exponential function** with base b is a function f defined by $f(x) = b^x$, $b > 0$, $b \neq 1$.
- The **logarithmic function** with base b , $b > 0$, $b \neq 1$, is the function defined by $f(x) = \log_b x$ such that $f(x) = y = \log_b x$ if and only if $x = b^y$, $x > 0$.
- The exponential and logarithmic functions are inverse functions of each other.
- The logarithm of a number to base b is the exponent to which the base b must be raised to get that number.

5. If x and y are positive real numbers, $b > 0$, $b \neq 1$, then
- $\log_b(xy) = \log_b x + \log_b y$
 - $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
 - $\log_b(x^r) = r \log_b x$ (r is a real number)
 - $\log_b 1 = 0$
 - $\log_b b = 1$
 - $b^{\log_b x} = x$
 - $\log_b b^x = x$
6. The **common logarithms** are logarithms with base 10, denoted by \log .
7. The **natural logarithms** are logarithms with base e , denoted by \ln .
8. $\log_a x = \frac{\log_b x}{\log_b a}$, $a > 0$, $b > 0$, $x > 0$, $a \neq 1$, $b \neq 1$.

Chapter 11 error analysis

1. Direct and inverse variation

Example: Given $P = \frac{kV}{T}$, $k > 0$, as the value of V

increases, the value of P will decrease, and as the value of T increases, the value of P will increase.

Correct answer: As V increases P will increase and as T increases P will decrease.

What error was made? (see page 470)

2. Exponential equations

Example: Find the solution set of the equation

$$4^x - 2 = 16^{5x+1}$$

$$4^x - 2 = (4^2)^{5x+1}$$

$$4^x - 2 = 4^{10x+2}$$

$$x - 2 = 10x + 2$$

$$-4 = 9x$$

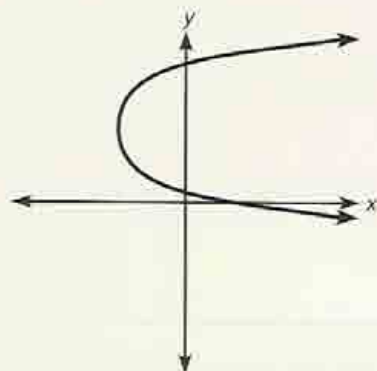
$$-\frac{4}{9} = x \quad \left\{ -\frac{4}{9} \right\}$$

$$\text{Correct answer: } \left\{ -\frac{2}{3} \right\}$$

What error was made? (see page 482)

3. Inverses of functions

Example: The graph represents a function f that has an inverse function.



Correct answer: The graph fails the vertical line test so does not represent a function and cannot be one-to-one. What error was made? (see page 462)

4. Logarithms

Example: The symbol " $\log_b y$ " is read "logarithm to the base y of b ."

Correct answer: "logarithm to the base b of y "

What error was made? (see page 485)

5. Equivalent logarithm statements

Example: The equation $81 = 3^4$ is equivalent to $\log_4 81 = 3$.

Correct answer: $\log_3 81 = 4$

What error was made? (see page 486)

6. Logarithms

Example: $\log_4 2 = \log_4(1 + 1) = \log_4 1 + \log_4 1$

Correct answer: $\log_4 2 = \frac{1}{2}$

What error was made? (see page 492)

7. Equivalent logarithm statements

Example: $\log_2 3 - \log_2 5 + 3 \log_2 4 = \log_2 \left(\frac{3 \cdot 3^4}{5} \right)$

$= \log_2 \left(\frac{3^5}{5} \right) = \log_2 \left(\frac{243}{5} \right)$

Correct answer: $\log_2 \left(\frac{192}{5} \right)$

What error was made? (see page 491)

8. Logarithmic equation solutions

Example: Find the solution set of

$$\log_4 x + \log_4(x - 15) = 2$$

$$\log_4 x(x - 15) = 2$$

$$x(x - 15) = 4^2$$

$$x^2 - 15x = 16$$

$$x^2 - 15x - 16 = 0$$

$$(x - 16)(x + 1) = 0$$

$$x - 16 = 0 \text{ or } x + 1 = 0$$

$$x = 16 \quad x = -1$$

$$\{16, -1\}$$

Correct answer: $\{16\}$

What error was made? (see page 492)

9. Exponential equations

Example: Find the solution set of $3^x = 7$ correct to four decimal places.

$$\log 3^x = \log 7$$

$$x \log 3 = \log 7$$

$$x = \frac{\log 7}{\log 3} = \log 7 - \log 3$$

$$= 0.8451 - 0.4771$$

$$= 0.3680$$

$$\{0.3680\}$$

Correct answer: $\{1.7713\}$

What error was made? (see page 505)

10. Squaring a radical binomial

Example:

$$(x - \sqrt{2x+1})^2 = (x)^2 - (\sqrt{2x+1})^2$$

$$= x^2 - 2x + 1$$

Correct answer: $x^2 - 2x\sqrt{2x+1} + 2x + 1$

What error was made? (see page 242)

Chapter 11 critical thinking

The epsilon fraternity has a ritualistic handshake that the members perform at the beginning and end of each meeting. If there are 20 members at a meeting, how many handshakes will be performed?

Note: This is combinations of 20 hands taken 2 at a time, but the answer could be found by logically thinking about the event.

Chapter 11 review

[11-1]

- Sketch the graph of the exponential function (a) $f(x) = 3^x$, (b) $g(x) = \left(\frac{1}{3}\right)^x$, (c) $h(x) = 4^{-x}$.
- Find the solution set of each exponential equation (a) $5^x = 625$, (b) $16^{2x-1} = 8$.

[11-2]

- Sketch the graph of $f(x) = \log_6 x$ by using the graph of $y = 6^x$.
- Write the logarithmic equation $\log_{1/3} 27 = -3$ as an exponential equation.
- Write the exponential equation $4^{-3} = \frac{1}{64}$ as a logarithmic equation.

Evaluate each logarithmic expression.

6. $\log_6 216$

7. $\log_{1/4} (256)$

8. $\log_9 \sqrt[3]{9}$

Find the solution set of each logarithmic equation.

9. $\log_x 125 = 3$

10. $\log_3 x = -5$

11. $\log_2 x = \frac{5}{2}$

12. For what values of x is $\log_{10}(x^2 + 2x - 15)$ defined? (*Hint:* $x^2 + 2x - 15 > 0$)

[11-3]

Use the properties of logarithms to write each logarithmic expression as a sum or difference of logarithms, or as a product of a real number times a logarithm. Use the prime factor form of each number. All variables are positive real numbers.

13. $\log_3 (56)$

14. $\log_4 \left(\frac{5}{6}\right)$

15. $\log_5 \left(\frac{9}{4}\right)$

Write each of the following logarithms as the logarithm of a single number. All variables are positive real numbers.

16. $\log_5 7 + \log_5 4$

17. $\log_6 15 - \log_6 3$

18. $4 \log_b x + 2 \log_b y - \log_b z$

19. $\log_4(x+3) - \log_4(x-4)$

20. $3 \log_b x + \log_b(2x-1) - 3 \log_b(x+1)$

Given $\log_b 2 = 0.3010$, $\log_b 3 = 0.4771$, and $\log_b 11 = 1.0413$, compute the following logarithms and then write the statement in equivalent exponential form. Round off to four decimal places.

21. $\log_b 22$

22. $\log_b 121$

23. $\log_b 96$

24. $\log_b \sqrt[3]{11}$

25. $\log_b \left(\frac{3}{22} \right)$

26. $\log_b \left(\frac{27}{\sqrt[4]{11}} \right)$

Find the solution set of each logarithmic equation.

27. $\log_4 x - \log_4 3 = 2$

28. $2 \log_2 3 + \log_2 x = 1$

29. $\log_5(2x+1) + \log_5 2 = 2$

30. $\log_3(x-3) - \log_3(x+2) = -3$

[11-4]

Find the following common logarithms.

31. $\log 342$

32. $\log 507,000$

33. $\log 0.00736$

34. Given the formula $\text{pH} = -\log[\text{H}^+]$, find pH when $[\text{H}^+] = 6.00312$.

35. The formula for compound interest is given by $A = P(1+i)^n$, where A is the amount after n years that the principal P will grow to at interest rate i percent (per year). Use common logarithms to find how many years it will take for the principal to triple ($A = 3P$) at 7.5% interest compounded annually. (Round off to the nearest tenth.)

[11-5]

Evaluate the following logarithms (correct to two decimal places) using $\log_a x = \frac{\log x}{\log a}$.

36. $\log_6 7$

37. $\log_{32} 4.73$

38. The number of a radioactive substance, present at time t , is given by $q = q_0 e^{-0.6t}$, where t is measured in days. What is the half-life of the substance? (Round to the nearest tenth.)

39. Using the formula in exercise 38, how long will it take for 2.3 grams of a substance to decay to 0.7 grams? (Round to the nearest tenth.)

[11-6]

Find the solution set of each exponential equation (correct to two decimal places).

40. $5^x = 17$

41. $4^{-x} = 3$

42. $15^{x-2} = 9$

43. $(4)^{2x} = 3$

44. $\left(\frac{4}{5} \right)^{3x} = 2$

45. A certain bacteria increases in number according to the equation $A = A_0(3)^{2t}$, where t is in hours, A_0 is the initial number, and A is the number of bacteria at time t . How long will it take for 1,000 bacteria to increase to 125,000 bacteria?

Chapter 11 cumulative test

- [1-1] 1. Given $\{x | -4 < x \leq 5\}$, list the integers in the set.

Perform the indicated operations and simplify.

[1-4] 3. $3[-6(9 - 5) - 3 \cdot 4 + 9]$

[3-2] 5. $(5x - 3y)(5x + 3y)$

Perform the indicated operations. Give the answer with positive exponents.

[3-1] 7. $(3a^2b)(-6a^3b^2)$

[3-3] 8. $(-3a^{-1}b^2c^{-3})^3$

[3-3] 9. $\left(\frac{4a^2b^{-3}}{12a^{-4}b^2}\right)^2$

Find the solution set of the following equations or inequalities.

[2-1] 10. $3(x + 2) - 4x = 5(2x - 1)$

[2-5] 12. $-5 \leq 2x - 1 < 4$

[2-5] 14. $|2 - 3x| \leq 4$

[6-1] 16. $2y^2 - y = 3$

[4-3] 18. Add $\frac{5}{4ab^2} + \frac{6}{8a^2b}$.

[4-2] 20. Divide $\frac{x^2 - 5x + 4}{x^2 - 25} \div \frac{x^2 - 4x + 3}{x^2 - 10x + 25}$.

[1-4] 2. Simplify the expression $4 - 12 \div 3 + 2^3 - 3^2 \cdot 4$.

[3-2] 4. $(2a - b)^2$

[3-2] 6. $(4y + 2)(3y - 5)$

[2-5] 11. $4y - 3 \leq 2y + 1$

[2-4] 13. $|4y - 3| = 2$

[2-5] 15. $|x + 5| > 6$

[4-7] 17. $\frac{3}{x} - \frac{2}{x} = \frac{4}{5}$

[4-3] 19. Subtract $\frac{6y}{y-7} - \frac{9y}{7-y}$.

[4-4] 21. Simplify the complex fraction $\frac{\frac{4}{x} - \frac{3}{y}}{\frac{4y - 3x}{xy}}$.

Perform the indicated operations and simplify.

[5-6] 22. $(2\sqrt{3} - 1)^2$

[5-7] 23. $(4 + 3i)(4 - 3i)$

[5-7] 24. i^{23}

[5-5] 25. $\sqrt{50} - 3\sqrt{8} + 2\sqrt{32}$

[5-6] 26. Rationalize the denominator of the expression $\frac{4}{\sqrt{10} - 2}$.

Find the solution set of the following equations.

[6-3] 27. $y^2 - 3y = 7$

[6-7] 29. Find the solution set of the inequality $z^2 - 3z \leq 40$. Graph the solution set on the number line.

[6-5] 28. $\sqrt{5x + 9} = x - 1$

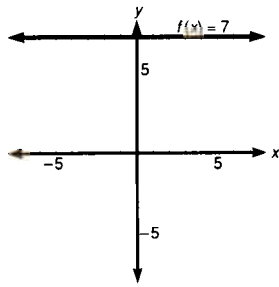
[7-3] 30. Find the equation of the line that satisfies the given conditions. Write the answer in standard form.
 (a) Passes through the points $(-1, 4)$ and $(0, 6)$
 (b) Passes through the point $(-9, 8)$ and parallel to $2x - 3y = 1$
 (c) Passes through $(5, -3)$ and having slope $\frac{4}{3}$

[10-2] 31. Given $f(x) = 5x + 1$ and $g(x) = x^3 + 2$, find
 (a) $f(5)$, (b) $g(-3)$, (c) $f[f(-2)]$,
 (d) $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$.

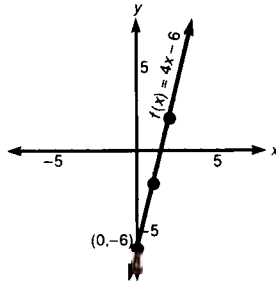
[7-3] 32. Sketch the graph of the equation $2x - 3y = -12$ using the slope and y -intercept.

- [9-1] 33. Sketch the graph of the equation
 $x = y^2 - 4y + 3$.
- [8-1] 35. Find the solution set of the system of linear equations
 $4x + 3y = 2$
 $2x - 5y = -1$.
- [11-3] 37. Find x , given (a) $\log_8 32 = 5$,
(b) $\log_6 x = -2$.
- [11-3] 39. Solve the equation
 $\log_3(x - 1) - \log_3(x - 2) = 2$.
- [9-3] 34. Determine if the graph of the given equation is a parabola, a circle, an ellipse, or a hyperbola:
(a) $3y^2 + 3x^2 = 14$, (b) $y^2 + 2y = x$,
(c) $4y^2 = 3x^2 + 9$, (d) $x^2 + 4y^2 = 4$.
- [11-1] 36. Find the solution set of the exponential equation
 $4^x = 16$.
- [11-5] 38. Evaluate $\log_7 14$ using natural logarithms and a calculator. (Round off to four decimal places.)
- [11-3] 40. Write the expression
 $\log_b 4 - 3 \log_b 5 + \log_b 2$ as the logarithm of a single expression.

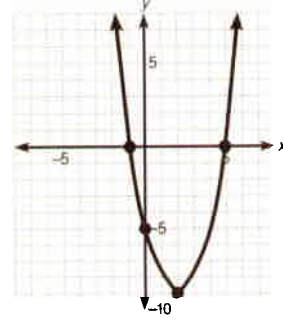
36. constant (linear)



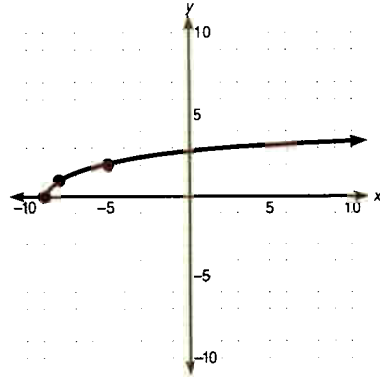
37. linear



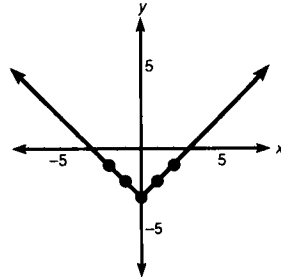
38. quadratic



39. square root



40. absolute value



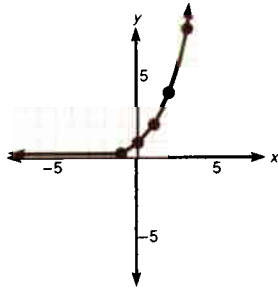
41. a. $k = 32$ b. $z = \sqrt[3]{4}$

Chapter 11

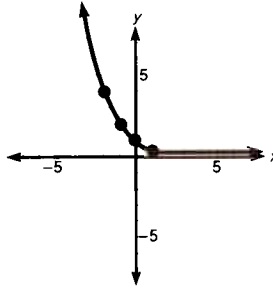
Exercise 11-1

Answers to odd-numbered problems

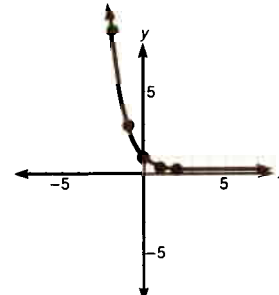
1.



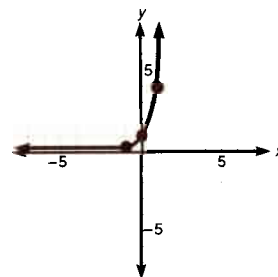
3.



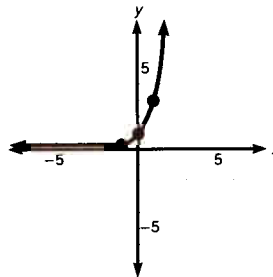
5.



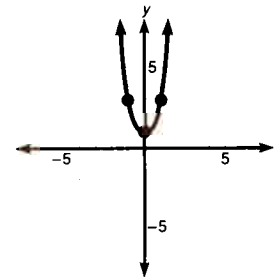
7.



9.



11.



13. $\{5\}$ 15. $\left\{\frac{5}{2}\right\}$ 17. $\left\{\frac{3}{2}\right\}$ 19. $\{-2\}$ 21. $\{-2\}$

23. $\left\{-\frac{5}{4}\right\}$ 25. $\left\{\frac{2}{9}\right\}$ 27. $\{-3\}$ 29. $\{3\}$ 31. $\{3\}$

33. $\left\{-\frac{1}{6}\right\}$ 35. a. 4,500 b. 40,500 c. 23,383

d. $1,500(3)^{1/12}$ 37. a. \$20,000 b. \$40,000 c. $5,000(2)^{1/9}$

39. a. 1,000 b. 250

Solutions to trial exercise problems5. Using $g(x) = 3^{-x}$ when

(1) $x = -2$,

$g(-2) = 3^{-(-2)} = 3^2 = 9$

$(-2, 9)$

(2) $x = -1$

$g(-1) = 3^{-(-1)} = 3$

$(-1, 3)$

(3) $x = 0$, $g(0) = 3^{-0} = 1$

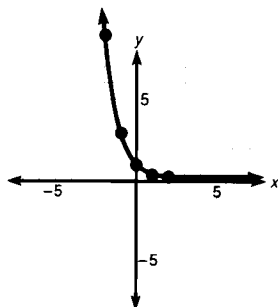
$(0, 1)$

(4) $x = 1$, $g(1) = 3^{-1} = \frac{1}{3}$

$\left(1, \frac{1}{3}\right)$

(5) $x = 2$, $g(2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$\left(2, \frac{1}{9}\right)$

10. $f(x) = 2^{x-1}$

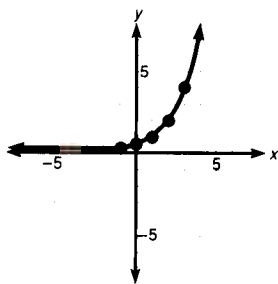
(1) $f(-1) = 2^{-1-1} = 2^{-2} = \frac{1}{4}$; $\left(-1, \frac{1}{4}\right)$

(2) $f(0) = 2^{-0-1} = 2^{-1} = \frac{1}{2}$; $\left(0, \frac{1}{2}\right)$

(3) $f(1) = 2^{1-1} = 2^0 = 1$; $(1, 1)$

(4) $f(2) = 2^{2-1} = 2^1 = 2$; $(2, 2)$

(5) $f(3) = 2^{3-1} = 2^2 = 4$; $(3, 4)$



15. $4^x = 32$. Since $4 = 2^2$ and $32 = 2^5$, then $(2^2)^x = 2^5$ and

$2^{2x} = 2^5$. Then $2x = 5$ and $x = \frac{5}{2}$. The solution set is $\left\{\frac{5}{2}\right\}$.

24. $9^{2x} = 27$. Since $9 = 3^2$ and $27 = 3^3$, then $(3^2)^{2x} = 3^3$ and

$3^{4x} = 3^3$. Thus $4x = 3$ and $x = \frac{3}{4}$. The solution set is $\left\{\frac{3}{4}\right\}$.

33. $27^{2x+1} = 9$. Since $27 = 3^3$ and $9 = 3^2$, then $(3^3)^{2x+1} = 3^2$

and $3^{6x+3} = 3^2$. Thus $6x + 3 = 2$, $6x = -1$, and $x = -\frac{1}{6}$.

The solution set is $\left\{-\frac{1}{6}\right\}$. 38. a. Let $t = 2$. Then $A =$

$200(3)^{-0.5(2)} = 200(3)^{-1} = \frac{200}{3} = 66\frac{2}{3}$ grams. b. Let $t = 12$.

Then $A = 200(3)^{-0.5(12)} = 200(3)^{-6} = \frac{200}{3^6} = \frac{200}{729}$ grams.

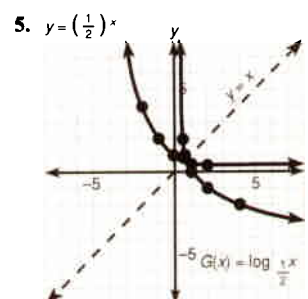
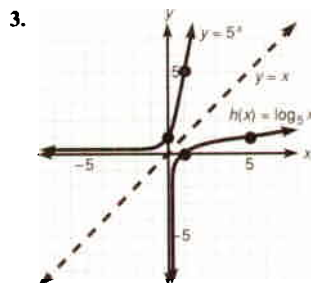
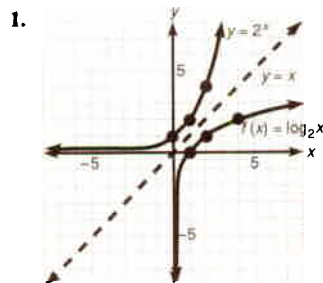
Review exercises

1. $f^{-1}(x) = \frac{x+3}{4}$ 2. domain = $\left\{x \mid x \in \mathbb{R}, x \neq \frac{5}{3}\right\}$

3. $f[g(x)] = 2x^2 + 3$ 4. $x^{1/6}$ 5. xy^3 6. $x^{2/3}$ 7. $\{-1, 5\}$

Exercise 11-2

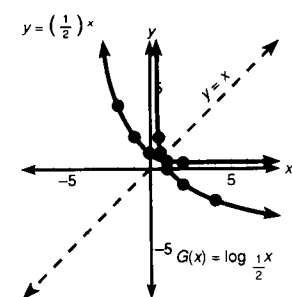
Answers to odd-numbered problems



7. $\log_3 81 = 4$ 9. $\log_2 64 = 6$ 11. $\log_{1/2} \left(\frac{1}{8}\right) = 3$
 13. $\log_{3/2} \left(\frac{8}{27}\right) = -3$ 15. $2^4 = 16$ 17. $10^3 = 1,000$
 19. $4^{-2} = \frac{1}{16}$ 21. $10^{-4} = 0.0001$ 23. $\left(\frac{1}{2}\right)^{-3} = 8$
 25. $\log_2 32 = 5$ 27. $\log_5 25 = 2$ 29. $\log_2 \left(\frac{1}{4}\right) = -2$
 31. $\log_6 \left(\frac{1}{216}\right) = -3$ 33. $\log_8 8 = 1$ 35. $\log_{1/3} \left(\frac{1}{81}\right) = 4$
 37. $\log_{5/4} \left(\frac{125}{64}\right) = 3$ 39. $\log_7 \sqrt[3]{7} = \frac{1}{3}$ 41. $\{2\}$ 43. $\{3\}$
 45. $\{7\}$ 47. $\{b | b > 0, b \neq 1\}$ 49. $\{81\}$ 51. $\left\{\frac{1}{36}\right\}$ 53. $\left\{\frac{1}{256}\right\}$
 55. $\left\{\frac{1}{16}\right\}$ 57. $\{p | p > -1\}$ 59. $\{y | y < -3 \text{ or } y > 4\}$

Solutions to trial exercise problems

5. Graph the equation $y = \left(\frac{1}{2}\right)^x$ and reflect this curve about the line $y = x$ to obtain the graph of $G(x) = \log_{1/2} x$.



11. $\frac{1}{8} = \left(\frac{1}{2}\right)^3$ is equivalent to $\log_{1/2} \left(\frac{1}{8}\right) = 3$.
 21. $\log_{10} 0.0001 = -4$ is equivalent to $10^{-4} = 0.0001$.
 29. $\log_2 \left(\frac{1}{4}\right) = x$ is equivalent to $2^x = \frac{1}{4}$. Since $\frac{1}{4} = \left(\frac{1}{2}\right)^2 = 2^{-2}$, then $2^x = 2^{-2}$ and $x = -2$. $\log_2 \left(\frac{1}{4}\right) = -2$.

34. $\log_{1/2} \left(\frac{1}{8}\right) = x$ is equivalent to $\left(\frac{1}{2}\right)^x = \frac{1}{8}$. Since $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, then $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^3$ and $x = 3$. $\log_{1/2} \left(\frac{1}{8}\right) = 3$.

43. $\log_x \left(\frac{1}{27}\right) = -3$ is equivalent to $x^{-3} = \frac{1}{27}$. Since $\frac{1}{27} = \frac{1}{3^3} = 3^{-3}$, then $x^{-3} = 3^{-3}$ and $x = 3$. The solution set is $\{3\}$.

48. $\log_{10} x = -2$ is equivalent to $10^{-2} = x$. Then $x = \frac{1}{100}$ and

the solution set is $\left\{\frac{1}{100}\right\}$. 56. Since $x - 2 > 0$ for $x > 2$, then $\log_6(x - 2)$ is defined for $\{x | x > 2\} = (2, +\infty)$.

Review exercises

1. $C = 25$ 2. $\frac{5-y}{2y+3}$ 3. $\frac{3z^2-5z-8}{z-2}$ 4. $\frac{1}{a^4c^{12}}$ 5. $-\frac{2}{3}$
 6. $\{0\}$

Exercise 11–3

Answers to odd-numbered problems

1. $\log_b 3 + \log_b 5$ 3. $\log_3 7 - \log_3 13$ 5. $3 \log_b 5$
 7. $4 \log_{10} 5$ 9. $2 \log_{1/2} 2 + 2 \log_{1/2} 3$ 11. $3 \log_5 2 + \log_5 3 - 2$
 13. $3(\log_b x + \log_b y)$ 15. $\frac{1}{2} \log_b x + \frac{1}{3} \log_b y$
 17. $\log_b(2x + 3y)$ 19. $\log_3 85$ 21. $\log_7 \frac{11}{16}$ 23. $\log_6 64$
 25. $\log_{10} 432$ 27. $\log_4 6$ 29. $\log_{10} \frac{200}{3}$ 31. $\log_b x^{2/3}$
 33. $\log_b \sqrt[4]{\frac{x^3}{y}}$ 35. $\log_5 \left(\frac{x^2 + 2x - 24}{9x^2} \right)$ 37. $\log_b(14) = 1.1461$;
 $b^{1.1461} = 14$ 39. $\log_b 49 = 1.6902$; $b^{1.6902} = 49$
 41. $\log_b(42) = 1.6232$; $b^{1.6232} = 42$ 43. $\log_b \left(\frac{7}{10} \right) = -0.1549$;
 $b^{-0.1549} = \frac{7}{10}$ 45. $\log_b \sqrt{7} = 0.4226$; $b^{0.4226} = \sqrt{7}$
 47. $\log_b \sqrt[3]{9} = 0.2386$; $b^{0.2386} = \sqrt[3]{9}$ 49. $\left\{ \frac{64}{5} \right\}$ 51. $\{20\}$
 53. $\{67\}$ 55. $\{3\}$ 57. $\{4\}$; -25 is extraneous 59. $\{3\}$; -9 is
 extraneous 61. $\left\{ \frac{-1 + \sqrt{33}}{4} \right\}$; $\frac{-1 - \sqrt{33}}{4}$ is extraneous
 63. $\left\{ \frac{-1 + \sqrt{5}}{2} \right\}$; $\frac{-1 - \sqrt{5}}{4}$ is extraneous 65. $\{1, 5\}$
 67. *Proof:* Let $\log_b u = m$ and $\log_b v = n$. Then $b^m = u$ and $b^n = v$.
 $\frac{u}{v} = \frac{b^m}{b^n} = b^{m-n}$. So $\log_b \frac{u}{v} = m - n = \log_b u - \log_b v$.
 69. Let $u = 1$, $v = 1$, and $b = 2$. Then
 (1) $\log_b(u + v) = \log_2(1 + 1) = \log_2 2 = 1$
 (2) $\log_b u = \log_2 1 = 0$; $\log_b v = \log_2 1 = 0$. Then $\log_2 1 + \log_2 1$
 $= 0$ and $\log_b(u + v) \neq \log_b u + \log_b v$.
- Solutions to trial exercise problems**
8. Since $24 = 2^3 \cdot 3$, then $\log_b(2^3 \cdot 3) = \log_b(2^3) + \log_b 3$
 $= 3 \log_b 2 + \log_b 3$. 10. Since $\frac{15}{16} = \frac{3 \cdot 5}{2^4}$, then $\log_{10} \left(\frac{15}{16} \right)$
 $= \log_{10} \left(\frac{3 \cdot 5}{2^4} \right) = \log_{10} 3 + \log_{10} 5 - \log_{10} 2^4 = \log_{10} 3 + \log_{10} 5$
 $- 4 \log_{10} 2$. 12. $\log_b \left(\frac{x^3}{y^2} \right) = \log_b x^3 - \log_b y^2 = 3 \log_b x - 2 \log_b y$
 13. $\log_b(xy)^3 = 3 \log_b(xy) = 3[\log_b x + \log_b y]$ 17. $\log_b(2x + 3y)$
 $= \log_b(2x + 3y)$, since none of the properties apply to addition.
 24. $2 \log_2 7 + \log_2 5 = \log_2 7^2 + \log_2 5 = \log_2 49 + \log_2 5$
 $= \log_2(49 \cdot 5) = \log_2(245)$ 28. $\log_5 4 + 2 \log_5 3 - 3 \log_5 2 = \log_5 4$
 $+ \log_5 3^2 - \log_5 2^3 = \log_5 4 + \log_5 9 - \log_5 8 = \log_5 \frac{4 \cdot 9}{8} = \log_5 \left(\frac{36}{8} \right)$
 $= \log_5 \left(\frac{9}{2} \right)$ 31. $\frac{1}{3} \log_b(x^2) = \log_b(x^2)^{1/3} = \log_b x^{2/3}$
 41. $\log_b(42) = \log_b(2 \cdot 3 \cdot 7) = \log_b 2 + \log_b 3 + \log_b 7 = (0.3010)$
 $+ (0.4771) + (0.8451) = 1.6232$. Then $b^{1.6232} = 42$.
 42. $\log_b \left(\frac{27}{4} \right) = \log_b 27 - \log_b 4 = \log_b 3^3 - \log_b 2^2 = 3 \log_b 3 - 2$
 $\log_b 2 = 3(0.4771) - 2(0.3010) = 1.4313 - 0.6020 = 0.8293$. Then
 $b^{0.8293} = \frac{27}{4}$. 54. $\log_4(x - 5) - \log_4 3 = 2$ is equivalent to
 $\log_4 \left(\frac{x - 5}{3} \right) = 2$, which is equivalent to $\frac{x - 5}{3} = 4^2$. Then $\frac{x - 5}{3} =$
 16 , $x - 5 = 48$, and $x = 53$. Thus the solution set is $\{53\}$.

57. $\log_{10}(x + 21) + \log_{10} x = 2$ is equivalent to $\log_{10} x(x + 21)$
 $= 2$. Then $x(x + 21) = 10^2$, $x^2 + 21x = 100$, and $x^2 + 21x - 100$
 $= 0$. Factoring, $(x + 25)(x - 4) = 0$, so $x = -25$ or $x = 4$. But if
 $x = -25$, then $\log_{10}(x + 21) = \log_{10}(-4)$, which does not exist.
 The solution set is $\{4\}$. (-25 is extraneous)

Review exercises

1. ellipse; x -intercepts, $(-3, 0)$, $(3, 0)$; y -intercepts, $(0, -\sqrt{3})$, $(0, \sqrt{3})$
 2. parabola; x -intercepts, $(-5, 0)$, $(4, 0)$; y -intercept, $(0, -20)$
 3. hyperbola; no x -intercepts; y -intercepts, $(0, -3)$, $(0, 3)$
 4. $\left\{ \frac{4}{3} \right\}$ 5. $\frac{4\sqrt{6}}{3}$ or $-\frac{4\sqrt{6}}{3}$ 6. 9 7. $\left\{ x \mid x \leq \frac{2}{5} \text{ or } x \geq \frac{6}{5} \right\}$
 $= \left(-\infty, \frac{2}{5} \right] \cup \left[\frac{6}{5}, \infty \right)$

Exercise 11–4

Answers to odd-numbered problems

1. $\log 8 = 0.9031$; $10^{0.9031} = 8$ 3. $\log(53) = 1.7243$; $10^{1.7243} = 53$
 5. $\log 80,200 = 4.9042$; $10^{4.9042} = 80,200$
 7. $\log 794,000,000 = 8.8998$; $10^{8.8998} = 794,000,000$ 9. $\log 0.00863$
 $= -2.0640$; $10^{-2.0640} = .00863$
 11. $\log 0.000000107 = -6.9706$; $10^{-6.9706} = 0.000000107$
 13. 5.7024 15. a. -4.1367
 b. 37.9912 17. a. 281.19 b. 10.99 19. 2.7773 21. 14.8532
 23. 34.7712 25. 6,990 27. 82,000,000 29. .0000165
 31. 338 33. .00589 35. .000000337 37. 48.3606
 39. 18.97 41. 3.3010 43. 1.585×10^{-6}

Solutions to trial exercise problems

1. $\log 8 = 0.9031$. Then $10^{0.9031} = 8$. 28. antilog -1.2472 .
 Press $\boxed{1} \boxed{0} \boxed{y^x} \boxed{1} \boxed{.} \boxed{2} \boxed{4} \boxed{7} \boxed{2} \boxed{+/-} \boxed{=}$
 and read “0.0565979 on the display. Antilog $-1.2472 = 0.0566$.
 30. Antilog 8.8340 is found by pressing $\boxed{1} \boxed{0} \boxed{y^x} \boxed{8} \boxed{.} \boxed{8} \boxed{3} \boxed{4} \boxed{0} \boxed{=}$. Read 6.823×10^8 . Then antilog 8.8340
 $= 682,300,000$. 43. $\text{pH} = -\log(\text{H}^+)$. Then $5.8 = -\log(\text{H}^+)$;
 $-5.8 = \log(\text{H}^+)$. Find antilog -5.8 by pressing $\boxed{1} \boxed{0} \boxed{y^x} \boxed{5} \boxed{.} \boxed{8} \boxed{+/-} \boxed{=}$. Read 0.000001585. Then $\text{H}^+ = 0.000001585$.

Review exercises

1. 14 2. $4ab - 3a - 5b$ 3. $\sqrt{2}$ 4. $-1 - 13i$
 5. $13\frac{1}{3}$ min 6. $66\frac{2}{3}$ cm³ of 10% solution; $33\frac{1}{3}$ cm³ of 4% solution

Exercise 11–5

Answers to odd-numbered problems

1. 1.2920 3. 2.3219 5. -2.6804 7. 1.2183 9. 1.6094
 11. 1.0043 13. 1.9066 15. -2.9878 17. 5.8493 19. 1.1333
 21. 2.9459 23. 4.0986 25. 13.7 27. 0.884 29. 0.0542
 31. 18.3 years 33. 55.9 years 35. 12,979.3 years
 37. 269.9 ft 39. 13.5 days

Solutions to trial exercise problems

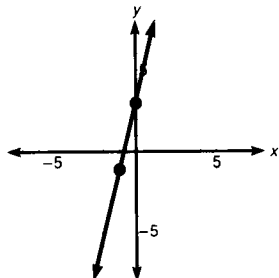
7. $\log_{23} 45.6 = \frac{\log 45.6}{\log 23} = \frac{1.6590}{1.3617} = 1.2183$
 10. $\log_e 3 = \ln 3$. Press $\boxed{3} \boxed{\ln}$ and read “1.0986” on the display.
 $\log_e 3 = 1.0986$ 12. $\log_e 107 = \ln 107$. Press $\boxed{1} \boxed{0} \boxed{7} \boxed{\ln}$ and
 read “4.6728” on the display. $\log_e 107 = 4.6728$

21. $\ln 7e = \ln 7 + \ln e = 1.9459 + 1 = 2.9459$ 37. $I = I_0 e^{-kd}$, given $k = 0.00853$, $I = 0.10I_0$, and we want d . Then $0.10I_0 = I_0 e^{-0.00853d}$; $0.10 = e^{-0.00853d}$; $-0.00853d = \ln 0.10$; $d = \frac{\ln 0.10}{-0.00853} = 269.9$. The light is reduced to 10% at a depth of 269.9 feet.

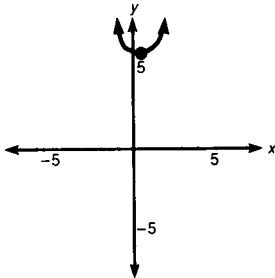
Review exercises

1. domain = $\{-4, 2, 0, 1\}$ 2. domain = $\left\{x \mid x \in \mathbb{R}, x \geq \frac{3}{2}\right\} = \left[\frac{3}{2}, \infty\right)$ 3. f is not one-to-one since ordered pairs $(-4, 3)$ and $(2, 3)$ have the same second component. 4. $\{(-1, -2), (2, 1)\}$

5.



6.



Exercise 11-6

Answers to odd-numbered problems

1. $\{3.17\}$ 3. $\{0.29\}$ 5. $\{-2.10\}$ 7. $\{1.19\}$
9. $\{0.26\}$ 11. $\{0.07\}$ 13. $\{1.71\}$ 15. $\{2.46\}$
17. $n = \frac{\log y}{3 \log x}$ 19. 4.2% 21. 3.9 years 23. 2.14 hours
25. 396 hours or approximately 24 minutes 27. 11.5 centuries
29. 2.05 years 31. 3.7%

Solutions to trial exercise problems

7. $6^{2x-1} = 12$; $\log 6^{2x-1} = \log 12$; $(2x-1)\log 6 = \log 12$; $2x \log 6 - \log 6 = \log 12$; $2x \log 6 = \log 12 + \log 6$; $x = \frac{\log 12 + \log 6}{2 \log 6} = \frac{1.0792 + 0.7782}{1.5564} = 1.19$. The solution set is $\{1.19\}$. 14. $2x^2 = 7$;

$\log 2x^2 = \log 7$; $x^2 \log 2 = \log 7$; $x^2 = \frac{\log 7}{\log 2} = \frac{0.8451}{0.3010} = 2.81$.

Then $x = \pm 1.68$. The solution set is $\{1.68, -1.68\}$. 17. $y = x^{3n}$;

$\log y = \log x^{3n}$; $\log y = 3n \log x$; $n = \frac{\log y}{3 \log x}$. 26. $q = q_0(0.96)^t$.

Now $q = \frac{1}{2}q_0 = 0.5q_0$; so $0.5q_0 = q_0(0.96)^t$; $0.5 = (0.96)^t$; $\log 0.5$

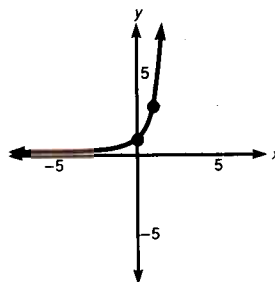
$= \log(0.96)^t$; $\log(0.5) = t \log(0.96)$; $t = \frac{\log(0.5)}{\log(0.96)} = \frac{-0.3010}{-0.0177} = 17.0$. The half-life of radium is 17.0 centuries.

Review exercises

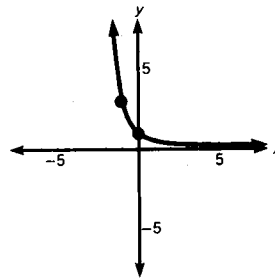
1. $(5x+4)(x-1)$ 2. $(5z+6)(5z-6)$ 3. $x(5x^2-2x+1)$
4. $(3z-2)^2$ 5. $\{-\sqrt{7}, \sqrt{7}, -i\sqrt{2}, i\sqrt{2}\}$ 6. $\{x \mid -5 \leq x < 2\}$
7. $d = \sqrt{101}$; midpoint, $\left(1, \frac{3}{2}\right)$

Chapter 11 review

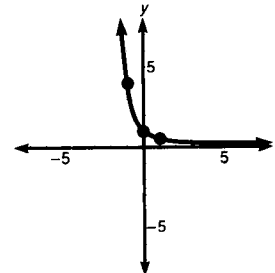
1. a.



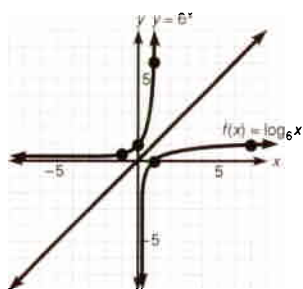
b.



c.



2. a. $\{4\}$ b. $\left\{\frac{7}{8}\right\}$ 3.



4. $\log_4 \frac{1}{64} = -3$ 5. $\left(\frac{1}{3}\right)^{-3} = 27$ 6. 3 7. -4 8. $\frac{1}{3}$

9. $\{5\}$ 10. $\left\{\frac{1}{243}\right\}$ 11. $\{4\sqrt{2}\}$ 12. $x < -5$ or $x > 3$

13. $\log_6 7 + 3 \log_6 2$ 14. $\log_4 5 - \log_4 2 - \log_4 3$

15. $2 \log_5 3 - 2 \log_5 2$ 16. $\log_5 28$ 17. $\log_6 5$ 18. $\log_6 \frac{x^4 y^2}{z}$

19. $\log_4 \left(\frac{x+3}{x-4}\right)$ 20. $\log_8 \left(\frac{x^3(2x-1)}{(x+1)^3}\right)$ 21. 1.3423;

$b^{1.3423} = 22$ 22. 2.0826; $b^{2.0826} = 121$ 23. 1.9821; $b^{1.9821} = 96$

24. 0.3471; $b^{0.3471} = \sqrt[3]{11}$ 25. -0.8652; $b^{-0.8652} = \frac{3}{22}$

26. 1.1710; $b^{1.1710} = \left(\frac{27}{\sqrt[4]{11}}\right)$ 27. $\{48\}$ 28. $\left\{\frac{2}{9}\right\}$ 29. $\left\{\frac{23}{4}\right\}$

30. $\left\{\frac{83}{26}\right\}$ 31. 2.5340 32. 5.7050 33. -2.1331

34. -0.7784 35. 15.2 yr 36. 1.09 37. 0.45 38. 1.2 days

39. 2.0 days 40. $\{1.76\}$ 41. $\{-0.79\}$ 42. $\{2.81\}$ 43. $\{0.40\}$

44. $\{-1.04\}$ 45. 2.20 hours

Chapter 11 cumulative test

1. $\{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$ 2. -28 3. -81

4. $4a^2 - 4ab + b^2$ 5. $25x^2 - 9y^2$ 6. $12y^2 - 14y - 10$

7. $-18a^5 b^3$ 8. $-\frac{27b^6}{a^3 c^9}$ 9. $\frac{a^{12}}{9b^{10}}$ 10. $\{1\}$ 11. $\{y|y \leq 2\}$

12. $\{x|-2 \leq x < \frac{5}{2}\}$ 13. $\left\{\frac{5}{4}, \frac{1}{4}\right\}$ 14. $\{x|-\frac{2}{3} \leq x \leq 2\}$

15. $\{x|x < -11$ or $x > 1\}$ 16. $\left\{\frac{3}{2}, -1\right\}$ 17. $\left\{\frac{5}{4}\right\}$

18. $\frac{5a+3b}{4a^2 b^2}$ 19. $\frac{15y}{y-7}$ 20. $\frac{x^2-9x+20}{x^2+2x-15}$ 21. 1

22. $13 - 4\sqrt{3}$ 23. 25 24. $-i$ 25. $7\sqrt{2}$ 26. $\frac{2\sqrt{10}+4}{3}$

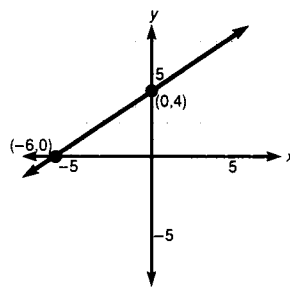
27. $\left\{\frac{3+\sqrt{37}}{2}, \frac{3-\sqrt{37}}{2}\right\}$ 28. $\{8\}$; -1 is extraneous

29. $\{z|-5 \leq z \leq 8\} = [-5, 8]$ 30. a. $2x - y = -6$

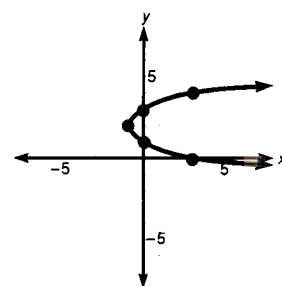
b. $2x - 3y = -42$ c. $4x - 3y = 29$ 31. a. 26 b. -25

c. -44 d. 5

32.



33.



34. a. circle b. parabola c. hyperbola d. ellipse

35. $\left\{\left(\frac{7}{26}, \frac{4}{13}\right)\right\}$ 36. $\{2\}$ 37. a. 2 b. $\frac{1}{36}$

38. 1.36 39. $\left\{\frac{17}{8}\right\}$ 40. $\log_6 \left(\frac{8}{125}\right)$

Chapter 12

Exercise 12-1

Answers to odd-numbered problems

1. 7, 11, 15, 19, 23 3. $\frac{2}{3}, \frac{1}{3}, \frac{2}{9}, \frac{1}{9}, \frac{2}{15}$ 5. $-6, \frac{7}{2}, \frac{8}{5}, \frac{9}{8}, \frac{10}{11}$

7. $\frac{2}{5}, \frac{2}{5}, \frac{8}{15}, \frac{4}{5}, \frac{32}{25}$ 9. -1, 7, -13, 19, -25 11. $1, -\frac{9}{5}, 3, -\frac{81}{17}, \frac{81}{11}$

13. 1, 1, 1, 1, 1 15. 32 17. $\frac{1}{111}$ 19. $\frac{65}{41}$ 21. 89

23. -247 25. $a_n = 2n + 4$ 27. $a_n = 5n - 3$

29. $a_n = n^3$ 31. $a_n = \frac{1}{2^n - 1}$ 33. $a_n = \frac{n+2}{2n+3}$

35. $a_n = (-1)^n(4n+2)$ 37. a. 27,000 b. 243,000

c. 1,000(3ⁿ) 39. $\frac{5}{2}$ ft; $\frac{5}{32}$ ft; $5\left(\frac{1}{2}\right)^{n-1}$ 41. a. \$16,000;

\$17,500; \$19,000; \$20,500; \$22,000; \$23,500

b. \$16,000 + 1500(n-1) c. \$44,500

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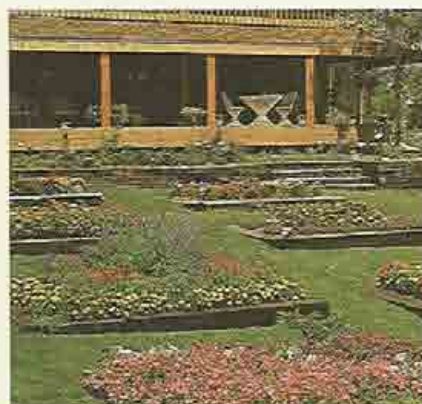
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